Monthly Past,
Monthly Present,
Monthly Future
(With Apologies to Charles Dickens)

Scott Chapman

Sam Houston State University

October 27, 2016
Some resources if you like this talk:


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A CONTRIBUTION TO THE MATHEMATICAL THEORY OF
BIG GAME HUNTING

H. PÉTARD, Princeton, New Jersey

This little known mathematical discipline has not, of recent years, received in the literature
the attention which, in our opinion, it deserves. In the present paper we present some algorithms
which, it is hoped, may be of interest to other workers in the field. Neglecting the more obviously
trivial methods, we shall confine our attention to those which involve significant applications
of ideas familiar to mathematicians and physicists.

The present time is particularly fitting for the preparation of an account of the subject, since
recent advances both in pure mathematics and in theoretical physics have made available powerful
tools whose very existence was unsuspected by earlier investigators. At the same time, some of the
more elegant classical methods acquire new significance in the light of modern discoveries. Like
many other branches of knowledge to which mathematical techniques have been applied in recent
years, the Mathematical Theory of Big Game Hunting has a singularly happy unifying effect on
the most diverse branches of the exact sciences.

For the sake of simplicity of statement, we shall confine our attention to Lions (*Felis leo*)
whose habitat is the Sahara Desert. The methods which we shall enumerate will easily be seen
to be applicable, with obvious formal modifications, to other carnivores and to other portions of
the globe. The paper is divided into three parts, which draw their material respectively from
mathematics, theoretical physics, and experimental physics.

The author desires to acknowledge his indebtedness to the Trivial Club of St. John's College,
Cambridge, England; to the M.I.T. chapter of the Society for Useless Research; to the F. o. P.,
of Princeton University; and to numerous individual contributors, known and unknown, con-
scious and unconscious.
Theorem 1. *There is a lion in the cage.*
Greetings from Huntsville!

Our Office

Big Sam
Some Basic Facts

Founded: 1896
Published Since 1915 by
The Mathematical Association of America
The Monthly is the most widely
Read Mathematics Journal in the World

Source: JSTOR and Ingenta data
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The most downloaded article (almost every month): The Problem Section.

A Monthly Problem led to Bill Gate’s only mathematical publication.


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Benjamin Finkel (1865–1947)  
Founder of the American Mathematical Monthly  
and Editor 1896–1912
“While realizing that the solution of problems is one of the lowest forms of Mathematical research, and that, in general, it has no scientific value, yet its educational value cannot be overestimated. . . . . The American Mathematical Monthly will, therefore, devote a due portion of its space to the solution of problems, whether they be the easy problems in Arithmetic, or the difficult problems in the Calculus, Mechanics, Probability, or Modern Higher Mathematics.”
L. E. Dickson (1874–1954)
Editor of the Monthly 1903–1906
In 1915 the American Mathematical Society, by a 3-2 Committee vote, decided not to take control of *The Monthly*. It did decide to lend support to any other organization which took over the journal. This led to the formation of the Mathematical Association of America, which still publishes *The Monthly*. 
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Some Notable Past Monthly Editors

Robert Carmichael (1918)

Herbert Wilf (1987-1991)

Paul Halmos (1982–1986)

Some Notable Monthly Authors

E. R. Hedrick   D. H. Lehmer   R. W. Hamming
L. R. Ford      Peter Sarnak   Andre Weil
I. Kaplansky    N. Bourbaki    Martin D. Kruskal
G. Pólya        William Feller E. T. Bell
C. L. Siegel    C. Fefferman   Walter Rudin
Walter Feit     Michael Atiyah Steve Smale
L. E. Dickson   George D. Birkhoff Saunders Mac Lane
Andrew Gleason  Felix Browder George Andrews
Phillip Griffiths Barry Mazur S. S. Chern
Stephen Wolfram Herbert Wilf David Eisenbud
John Conway     Joseph Silverman Branko Grünbaum
Most Cited Articles

Most Cited Articles according to Google Scholar (as of October 25, 2016)


5. Cited 1612 times: *Can One Hear the Shape of a Drum?*, Mark Kac, 73(1966), 1-23.
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Lloyd Shapley delivers his Nobel Prize Lecture
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How Tokieda Sees It

The Special Issue includes the Ford-Halmos Award winning paper *Roll Models* by Tadashi Tokieda.

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Pi Day Is Upon Us Again and We Still Do Not Know if Pi Is Normal

David H. Bailey and Jonathan Borwein

Abstract. The digits of π have intrigued both the public and research mathematicians from the beginning of time. This article briefly reviews the history of this venerable constant, and then describes some recent research on the question of whether π is normal, or, in other words, whether its digits are statistically random in a specific sense.

1. PI AND ITS DAY IN MODERN POPULAR CULTURE. The number π, unique among the pantheon of mathematical constants, captures the fascination both of the public and of professional mathematicians. Algebraic constants such as √2 are easier to explain and to calculate to high accuracy (e.g., using a simple Newton iteration scheme). The constant e is pervasive in physics and chemistry, and even appears in financial mathematics. Logarithms are ubiquitous in the social sciences. But none of these other constants has ever gained much traction in the popular culture.

In contrast, we see π at every turn. In an early scene of Ang Lee’s 2012 movie adaptation of Yann Martel’s award-winning book *The Life of Pi*, the title character
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It contains papers from some of the leading people in the field including Mike Steel, Mark Lewis, David Terman, Trachette Jackson, and Jeffrey Poet
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Some Other Papers of Present and Future Interest

The First 100 Years of the MAA
David E. Zitarelli

Abstract. Why was the MAA founded? What role has the Association played in American mathematics? What were its primary activities? We answer these questions in this overview of the MAA over its 100-year history from its founding in 1915. Along the way, we describe MAA sections, governance, meetings, prizes/awards, and headquarters. The account of MAA activities is divided into two periods, 1916–1955 and 1955–2014 and contains a discussion for the critical role played by the Committee on the Undergraduate Program in Mathematics in this division.

1. INTRODUCTION. This article presents an overview of the history of the Mathematical Association of America as part of the celebration of its centennial in 2015. It describes events this author regards as the most important over the century, but the account is certainly not exhaustive; for example, it makes little mention of competitions conducted under the aegis of the Association or of the expanded book publication program. Our account begins with the founding of the MAA and then describes its sections, governance, and meetings. Overarching activities are outlined in two distinct periods, 1916–1955 and 1955–2014, and I supply an explanation for the partition into disjoint stages. The article then discusses prizes and awards before ending with a brief mention of MAA headquarters.
I Prefer Pi: A Brief History and Anthology of Articles in the American Mathematical Monthly

Jonathan M. Borwein and Scott T. Chapman

Abstract. In celebration of both a special “big” π Day (3/14/15) and the 2015 centennial of the Mathematical Association of America, we review the illustrious history of the constant π in the pages of the American Mathematical Monthly.

1. INTRODUCTION. Once in a century, Pi Day is accurate not just to three digits but to five. The year the MAA was founded (1915) was such a year and so is the MAA’s centennial year (2015). To arrive at this auspicious conclusion, we consider the date to be given as month–day–two-digit year. This year, Pi Day turns 26. For a more detailed discussion of Pi and its history, we refer to last year’s article [46]. We do note that “I prefer π” is a succinct palindrome.²

In honor of this happy coincidence, we have gone back and selected roughly 76 representative papers relating to Pi (the constant not the symbol) published in this journal since its inception in 1894 (which predates that of the MAA itself). Those 75 papers listed in three periods (before 1945, 1945–1989, and 1990 on) form the core bibliography of this article. The first author and three undergraduate research students³ ran a seminar in which they looked at the 75 papers. Here is what they discovered.
Here are some titles that will round out 2016.

5. *Anonymity in Predicting the Future*, by D. Bajpai and Dan Velleman.
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Factorizations of Algebraic Integers, Block Monoids, and Additive Number Theory

Paul Baginski and Scott T. Chapman

Abstract. Let $D$ be the ring of integers in a finite extension of the rationals. The classic examination of the factorization properties of algebraic integers usually begins with the study of norms. In this paper, we show using the ideal class group, $C(D)$, of $D$ that a deeper examination of such properties is possible. Using the class group, we construct an object known as a block monoid, which allows us to offer proofs of three major results from the theory of nonunique factorizations: Geroldinger’s theorem, Carlitz’s theorem, and Valenza’s theorem. The combinatorial properties of block monoids offer a glimpse into two widely studied constants from additive number theory, the Davenport constant and the cross number. Moreover, block monoids allow us to extend these results to the more general classes of Dedekind domains and Krull domains.

1. INTRODUCTION. In an introductory abstract algebra class, the notion of a unique factorization domain (UFD) is carefully developed and plays an important role. A wide array of UFDs are usually identified in such a course (such as $\mathbb{Z}$, $K[X]$ where $K$ is a field, and $\mathbb{Z}[i]$, the Gaussian integers) before deeper algebraic structures, such as Euclidean domains or principal ideal domains, are introduced. To convince a student of the usefulness of the definition of a UFD (also known as a factorial domain), it is necessary to provide an example of an integral domain in which the notion of unique factorization fails. While there is an abundance of such examples, the one

...
The Setting

Let

\[ \mathbb{Z} = \{ \ldots -3, -2, -1, 0, 1, 2, 3, \ldots \} \]

represent the integers

\[ \mathbb{N} = \{ 1, 2, 3, \ldots \} \]

represent the natural numbers and

\[ \mathbb{N}_0 = \{ 0, 1, 2, \ldots \} \]

represent the natural numbers adjoin 0.
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This part of the talk will be based on the simple congruence relation on $\mathbb{Z}$ defined by

$$a \equiv b \pmod{n}$$

if and only if

$$n \mid a - b \text{ in } \mathbb{Z}$$
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Why we love $a \equiv b \pmod{n}$

One of our basic arithmetic operations work well here:

If $a \equiv b \pmod{n}$ and $c \in \mathbb{Z}$, then

$$ca \equiv cb \pmod{n}$$

BUT

$ca \equiv cb \pmod{n}$ does not imply that $a \equiv b \pmod{n}$. 
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Let’s consider a very simple arithmetic sequence:

\[1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49 = \{1 + 4k \mid k \in \mathbb{N}_0\} = 1 + 4\mathbb{N}_0\]

is known as The Hilbert Monoid.
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What is a Monoid?

What is a monoid?

A set $S$ with a binary operation $\ast$ (like $+$ or $\times$ on the real numbers) which satisfies the following.

1. $\ast$ is closed on $S$.
2. $\ast$ is associative on $S$ $((a \ast b) \ast c = a \ast (b \ast c))$
3. $\ast$ has an identity element $e$ $(a \ast e = e \ast a)$

Examples: $\mathbb{Z}$, $\mathbb{R}$ or $\mathbb{Q}$ under $\ast$. 
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**Examples:** $\mathbb{Z}$, $\mathbb{R}$ or $\mathbb{Q}$ under $+$. 
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David Hilbert
(1912)
In $1 + 4\mathbb{N}_0$ we have

$$21 \cdot 33 = 9 \cdot 77$$

$$(3 \cdot 7) \cdot (3 \cdot 11) = (3 \cdot 3) \cdot (7 \cdot 11)$$

and clearly 9, 21, 33 and 77 cannot be factored in $1 + 4\mathbb{N}_0$. But notice that 9, 21, 33 and 77 are not prime in the usual sense of the definition in $\mathbb{Z}$. 
Hilbert’s Argument

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Let $D = \mathbb{Z}[\sqrt{-5}] = \{a + b\sqrt{-5} \mid a, b \in \mathbb{Z}\}$.

In $D$

$$6 = 2 \cdot 3 = (1 + \sqrt{-5})(1 - \sqrt{-5})$$

represents a nonunique factorization into products of irreducibles in $D$. To fully understand this, a student must understand *units* and *norms* in $D$. 
Let \( D = \mathbb{Z}[\sqrt{-5}] = \{ a + b\sqrt{-5} \mid a, b \in \mathbb{Z} \} \).

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Let $M$ be a monoid. Call $x \in M$

(1) *prime* if whenever $x \mid yz$ for $x$, $y$, and $z$ in $M$, then either $x \mid y$ or $x \mid z$.

(2) *irreducible (or an atom)* if whenever $x = yz$ for $x$, $y$, and $z$ in $M$, then either $y \in M^\times$ or $z \in M^\times$.

As usual, 

$$x \text{ prime in } M \implies x \text{ irreducible in } M$$

but not conversely.
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Lemma

The element $x$ is irreducible in $1 + 4\mathbb{N}_0$ if and only if $x$ is either
1. $p$ where $p$ is a prime and $p \equiv 1 \pmod{4}$, or
2. $p_1p_2$ where $p_1$ and $p_2$ are primes congruent to 3 modulo 4.

Moreover, $x$ is prime if and only if it is of type 1.

Corollary

Let $x \in 1 + 4\mathbb{N}_0$. If

$$x = \alpha_1 \cdots \alpha_s = \beta_1 \cdots \beta_t$$

for $\alpha_i$ and $\beta_j$ in $A(1 + 4\mathbb{N}_0)$, then $s = t$. 
Lemma

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Corollary

Let $x \in 1 + 4\mathbb{N}_0$. If

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for $\alpha_i$ and $\beta_j$ in $\mathcal{A}(1 + 4\mathbb{N}_0)$, then $s = t$. 
An Example to Illustrate the Last Two Results

Let’s Factor

$$141,851,281 = 4 \times (35,462,820) + 1 \in 1 + 4\mathbb{N}_0$$

Now

$$141,851,281 = 13 \times 17 \times 11 \times 23 \times 43 \times 59$$
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Conclusion of $1 + 4\mathbb{N}_0$

In general, a monoid with this property, i.e.,

$$x = \alpha_1 \cdots \alpha_s = \beta_1 \cdots \beta_t$$

for $\alpha_i$ and $\beta_j$ in $A(M)$, then $s = t$, is called *half-factorial*.

**Theorem**

There is a map

$$\varphi : \mathbb{Z}[\sqrt{-5}] \to 1 + 4\mathbb{N}_0$$

which preserves lengths of factorizations into products of irreducibles. Hence, $\mathbb{Z}[\sqrt{-5}]$ is half-factorial.
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Arrangements of Stars on the American Flag

Dimitris Koukoulopoulos and Johann Thiel

Abstract. In this article, we examine the existence of nice arrangements of stars on the American flag. We show that despite the existence of such arrangements for any number of stars from 1 to 100, with the exception of 29, 69 and 87, they are rare as the number of stars increases.

1. INTRODUCTION. The Union Jack is the blue, upper left portion of the American flag that contains a star for each state in the Union\(^1\). From 1777 to 2002, the Union Jack was the maritime flag used by American Navy vessels to establish their nationality.

With the passage of the Puerto Rico Democracy Act by Congress in 2010, the door has been opened for Puerto Rico to set a referendum which could result in it becoming the 51st state. By law, each time a state is admitted into the Union, the government must create a new Union Jack with an extra star by the fourth of July following the admittance of the new state. In a recent Slate article [8], Chris Wilson considered this possibility and raised the question: What might a 51 star Union Jack look like?

The current 50 star jack design is usually credited to Robert G. Heft [7]. In 1958, when the debate over Alaska’s statehood status was being considered, Heft claims to have guessed correctly that Congress would eventually allow Alaska to join the Union only if Hawaii was admitted as well. Back then Alaska was Democrat territory, while Hawaii was more Republican, so admitting both would maintain the current balance of power in Congress. While some designers were working on a 49 star arrangement, Heft had already created a 50 star arrangement one year ahead of Hawaii becoming
(a) **Long** - This pattern is made by alternating long and short rows of stars where the long rows contain one more star than the short rows. In this pattern the first and last row are both long.

(a) Long (50 stars, 1960)
(b) **Short** - This pattern is defined in the same way as the long pattern, except that the first and the last row are both short.
(c) **Alternate** - This pattern is defined in the same way as the long pattern, except that the first row is long and the last row is short, or vice versa.

(c) Alternate (45 stars, 1896)
(d) **Wyoming** - In this pattern, the first and the last row are long, while all other rows are short.
**Equal** - All rows in this pattern have the same number of stars.

(e) Equal (48 stars, 1912)
(f) Oregon - Similar to the equal pattern, all rows in this pattern are of the same length with the exception of the middle row, which is two stars shorter. This pattern requires an odd number of rows.
Configurations of Stars on the Flag

(g) Long (50 stars, 1960)

(h) Short

(i) Alternate (45 stars, 1896)

(j) Wyoming (32 stars, 1858)

(k) Equal (48 stars, 1912)

(l) Oregon (33 stars, 1859)
**The Odd Person Out**

**Figure:** 29 star Union Jack

**Definition**
A *nice* arrangement of stars on the Union Jack is one that uses one of the six patterns defined above with a column to row ratio in the interval $[1, 2]$.

**Fact:** All integers from 1 to 100, except for 29, 69 and 87, have at least one nice arrangement.
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A Necessary Deep Theorem

Let

\[ \Omega(n) = \text{the number of prime factors of } n \text{ counted with multiplicity.} \]

**Theorem (Hardy, Ramanujan)**

For any fixed \( \epsilon > 0 \) we have that

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\lim_{N \to \infty} \frac{1}{N} \# \{ n \leq N : (1 - \epsilon) \log \log N \leq \Omega(n) \leq (1 + \epsilon) \log \log N \} = 1.
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Let

\[ S(N) = \#\{n \leq N : \text{there is a nice arrangement of } n \text{ stars on the Union Jack}\}. \]

By previous computation, \( S(100) = 97 \).

**Theorem**

\[ \lim_{N \to \infty} \frac{S(N)}{N} = 0. \]
The Surprising Conclusion

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