Worksheet 4.3A, The Sine Wave
MATH 1410
(SOLUTIONS)

1. Graph the following functions in the interval $[-2\pi, 2\pi]$ and then carefully describe the transformations necessary to change the graph of $y = \sin x$ into the graph of

   (a) $y = \sin(x + 1)$
   (b) $y = \sin x + 1$
   (c) $y = \sin 2x$
   (d) $y = 2 \sin x$
   (e) $y = 3 \sin(2x - \frac{\pi}{2})$
   (f) $y = 5 \sin 2x + 1$
   (g) $y = 3 \sin(4x - \pi) - 3$
   (h) $y = -5 \sin(2(x - \frac{\pi}{2})) + 1$

   Solutions.

   (a) To graph $y = \sin(x + 1)$, take the graph of $y = \sin x$ and
   shift it one unit to the left.

   (b) To graph $y = \sin x + 1$, take the graph of $y = \sin x$ and
   shift it up one unit.

   (c) To graph $y = \sin 2x$, take the graph of $y = \sin x$ and
   shrink it horizontally by a factor of 2.

   (d) To graph $y = 2 \sin x$, take the graph of $y = \sin x$ and
   stretch it vertically by a factor of 2.

   (e) To graph $y = 3 \sin(2x - \frac{\pi}{2})$, first note that $2x - \frac{\pi}{2} = 2(x - \frac{\pi}{4})$ and so take the graph of $y = \sin x$ and
   (1) shift $\frac{\pi}{4}$ to the right, (2) shrink horizontally by a factor of 2 centered on $(\pi/4, 0)$, and then
   (3) stretch vertically be factor of 3.

   (f) To graph $y = 5 \sin 2x + 1$, take the graph of $y = \sin x$ and
   (1) shrink horizontally by a factor of 2, (2) stretch vertically be factor of 5, (3) shift up 1.

   (g) To graph $y = 3 \sin(4x - \pi) - 3$, note that $4x - \pi = 4(x - \pi/4)$, and so take the graph of $y = \sin x$ and
   (1) shift $\frac{\pi}{4}$ to the right, (2) shrink horizontally by a factor of 3 centered on $(\frac{\pi}{4}, 0)$
   (3) stretch vertically by factor of 3 *and* and then (4) shift down 3.

   (h) To graph $y = -5 \sin(2(x - \frac{\pi}{2})) + 1$, take the graph of $y = \sin x$ and
   (1) shift $\frac{\pi}{2}$ to the right, (2) shrink horizontally by a factor of 2 centered on $(\frac{\pi}{2}, 0)$
   (3) stretch vertically by factor of 5 *and* reflect across the y-axis and then (4) shift up 1.

2. For each of the eight functions below, give the amplitude, period and phase shift. (These are the same functions found in problem 1.)

   (a) $y = \sin(x + 1)$
   (b) $y = \sin x + 1$
   (c) $y = \sin 2x$
   (d) $y = 2\sin x$
   (e) $y = 3\sin(2x - \frac{\pi}{2})$
   (f) $y = 5\sin 2x + 1$
   (g) $y = 3\sin(4x - \pi) - 3$
   (h) $y = -5\sin(2(x - \frac{\pi}{2})) + 1$

   Solutions.

   (a) $y = \sin(x + 1)$ has amplitude 1, period 2π and phase shift -1.
   (b) $y = \sin x + 1$ has amplitude 1, period 2π and phase shift 0.
   (c) $y = \sin 2x$ has amplitude 1, period π and phase shift 0.
   (d) $y = 2\sin x$ has amplitude 2, period 2π and phase shift 0.
   (e) $y = 3\sin(2x - \frac{\pi}{2})$ has amplitude 3, period π and phase shift $\frac{\pi}{4}$. (Notice that the phase shift is *not* $\frac{\pi}{2}$ since $2x - \frac{\pi}{2} = 2(x - \frac{\pi}{4})$.)
   (f) $y = 3\sin(4x - \pi) - 3$ has amplitude 3, period $\pi/2$ and phase shift $\pi/4$.
   (g) $y = 5\sin 2x + 1$ has amplitude 5, period π and phase shift 0.
   (h) $y = -5\sin(2(x - \frac{\pi}{2})) + 1$ has amplitude 5, period π and phase shift $\frac{\pi}{2}$.

3. Let $f(x) = 4\sin(2x - \pi) - 10$.

   (a) Find the domain of $f(x)$.
   (b) Find the range of $f(x)$.
   (c) Find the amplitude, period, frequency and phase shift of the function $f(x)$
   (d) Describe, in order, the transformations used to move the graph of $y = \sin x$ onto the graph of $y = f(x)$.

   Solutions.

   (a) Every real number can be an input into the sine function. So the domain of $f(x)4\sin(2x - \pi) - 10$ is $\boxed{(-\infty, \infty).}$
   (b) The range of the sine function is $[-1, 1]$. Here we multiply by 5 and then subtract 10 so the range is $\boxed{[-15, -5].}$
   (c) Simple algebra allows us to rewrite $f(x) = 4\sin(2(x - \frac{\pi}{2})) - 10$. $\boxed{\text{The period is } 2\pi/2 = \pi, \text{ the amplitude is } 4,}$ $\boxed{\text{the frequency is } \frac{1}{\pi} \text{ and the phase shift is } -\pi/2.}$
   (d) The transformations used to move the graph of $y = \sin x$ onto the graph of $f(x) = 4\sin(2x - \pi) - 10$ are, in this order,
      i. Shift right by $\pi/4$,
      ii. Shrink horizontally by a factor of 2 (centered on $(\pi/4, 0)$)
      iii. Expand vertically by a factor of 4,

iv. Shift down 10.

4. A ball bouncing on a string has height given by the equation

$$h(t) = 6\sin\frac{\pi}{4}t + 10$$

where t is measured in seconds and h is measured in inches.

(a) Find the amplitude and period of $h(t)$.

(b) What are the maximum and minimum heights of the ball as it bounces?

(c) Find the frequency of $h(t)$. Interpret the frequency: what does this tell us about the number of bounces of the ball?

(d) Graph $h(t)$ over the interval $[0, 8]$.

Partial solutions.

(a) The amplitude of $h(t)$ is 6 and period of $h(t)$ is 8.

(b) The ball bounces between $y = 16$ and $y = 4$.

(c) The frequency of the bouncing ball is $\frac{1}{8}$. This says that in one second the ball goes through one-eighth of a full bound.

(d) Over the interval $[0, 8]$ the graph goes through one sine wave; it takes 8 seconds for the ball to bounce.

5. A person's blood pressure follows a sine wave corresponding to the beat's of the heart. Suppose a particular individual's blood pressure at time t (measured in minutes) is

$$p(t) = 20\sin(160\pi t) + 100.$$

(a) Graph $p(t)$ in the interval $[0, 0.1]$.

(b) What is the maximum blood pressure for this person?

(c) What is the minimum blood pressure for this person?

(d) What is this person's heart rate?

Partial solutions.

(a) Over interval $[0, 0.1]$ the graph repeats 8 sine waves.

(b) & (c) The maximum blood pressure for this person is 120 while the minimum is 80.

(d) Since the period of the sine wave is $\frac{2\pi}{160\pi} = \frac{1}{80}$, the frequency is 80. This means that this person's heart beats 80 times in one minute.

6. The voltage $V(t)$ in an electrical circuit follows the equation

$$V(t) = 156\cos 120\pi t$$

where t is measured in seconds.

(a) Find the amplitude and period of $V(t)$.

(b) Find the frequency of $V(t)$.

(c) Graph $V(t)$ over the interval $[0, 0.1]$.

Partial solutions. The amplitude of $V(t)$ is 156 while the period is $\frac{2\pi}{120\pi} = \frac{1}{60}$. The frequency of the current is 60 cycles a second (that is, 60 Hertz). Over the interval $[0, 0.1]$ the sine wave should repeat six times.