Worksheet 2.1A, Quadratic functions
MATH 1410,
(SOLUTIONS)

1. Find the quadratic function with the given vertex and point. Put your answer in standard form.
   (a) Vertex $(0, 0)$ passing through $(-2, 8)$.
   (b) Vertex $(2, 0)$ passing through $(1, 3)$.
   (c) Vertex $(-3, 0)$ passing through $(-5, -4)$.
   (d) Vertex $(0, 1)$ passing through $(-1, 0)$.
   (e) Vertex $(2, 5)$ passing through $(3, 7)$.
   (f) Vertex $(-3, 4)$ passing through $(0, 0)$.

   Solutions.

   (a) A parabola with vertex $(0, 0)$ passing through $(-2, 8)$ has equation $\boxed{y = 2x^2.}$
   (b) A parabola with vertex $(2, 0)$ passing through $(1, 3)$ has equation $\boxed{y = 3(x-2)^2.}$
   (c) A parabola with vertex $(-3, 0)$ passing through $(-5, -4)$ has equation $\boxed{y = -(x+3)^2.}$
   (d) A parabola with vertex $(0, 1)$ passing through $(-1, 0)$ has equation $\boxed{y = -x^2 + 1.}$
   (e) A parabola with vertex $(2, 5)$ passing through $(3, 7)$ has equation $\boxed{y = 2(x-2)^2 + 5.}$
   (f) A parabola with vertex $(-3, 4)$ passing through $(0, 0)$ has equation $\boxed{y = -\frac{4}{9}(x+3)^2 + 4.}$

2. In each problem below, complete the square of the expression in x on the right-hand side of the equation. Then use that work to find the vertex of the graph of the quadratic function.

   (a) $y = x^2 + 4x$
   Solution.
   Completing the square on $x^2 + 4x$ gives $\boxed{x^2 + 4x = (x+2)^2 - 4.}$
   Therefore the graph $y = (x+2)^2 + 4$ has vertex $\boxed{(-2, -4).}$

   (b) $y = x^2 - 2x + 2$
   Solution.
   Completing the square on $x^2 - 2x$ gives $x^2 - 2x = (x-1)^2 - 1$ so $\boxed{x^2 - 2x + 2 = (x-1)^2 + 1.}$
   Therefore the graph $y = (x-1)^2 + 1$ has vertex $\boxed{(1, 1).}$

   (c) $y = 6x - 10 - x^2$
   Solution.
   To complete the square on $-x^2 + 6x - 10 = -(x^2 - 6x) - 10$ note that $x^2 - 6x = (x-3)^2 - 9$ so $-(x^2 - 6x) - 10 = -((x-3)^2 - 9) - 10 = -(x-3)^2 + 9 - 10 = \boxed{-(x-3)^2 - 1.}$
   The graph $y = -(x-3)^2 - 1$ has vertex $\boxed{(3, -1).}$

(d) $y = 8 + 3x - x^2$

Solution.

To complete the square on $8+3x-x^2$, first factor out -1 and write $8+3x-x^2 = -(x^2-3x-8)$. Note that $x^2 - 3x + \frac{9}{4} = (x - \frac{3}{2})^2$ so that $x^2 - 3x - 8 = (x - \frac{3}{2})^2 - 8 - \frac{9}{4} = (x - \frac{3}{2})^2 - \frac{41}{4}$.

The graph of $y = -[(x - \frac{3}{2})^2 - \frac{41}{4}]$ opens downward and has vertex $\boxed{(\frac{3}{2}, \frac{41}{4}).}$

(e) $y = 2x^2 - 8x + 9$

Solution.

To complete the square on $2x^2 - 8x + 9$, first factor out the 2 and write $2(x^2 - 4x) + 9$ and observe that $x^2 - 4x = (x - 2)^2 - 4$.

So $2(x^2 - 4x) + 9 = 2((x - 2)^2 - 4) + 9 = 2(x - 2)^2 - 8 + 9 = \boxed{2(x - 2)^2 + 1.}$

The graph of $y = 2(x - 2)^2 + 1$ has vertex $\boxed{(2, 1).}$

3. Put the function in standard form $y = a(x - h)^2 + k$ and then describe the transformation required to graph this function beginning with the graph of $y = x^2$.

(a) $y = x^2 + 4x$

(b) $y = x^2 - 2x + 2$

(c) $y = 6x - 10 - x^2$

(d) $y = 8 + 3x - x^2$

(e) $y = 2x^2 - 8x + 9$

Solution.

(a) From a previous problem we see that graph of $y = (x + 2)^2 - 4$ has vertex $(-2, -4)$. So we first shift the graph of $y = x^2$ left by 2 and then up by 4.

(b) From the previous problem we see that graph of $y = (x - 1)^2 + 1$ has vertex $(1, 1)$. So we first shift the graph of $y = x^2$ right by 1 and then up by 1.

(c) From the previous problem we see that graph of $y = (x - 3)^2 - 1$ has vertex $(3, -1)$. So we first shift the graph of $y = x^2$ right by 3, reflect it across the x axis (due to the leading coefficient -1) and then shift down by 1.

(d) From the previous problem we see that graph of $y = -[(x - \frac{3}{2})^2 - \frac{41}{4}] = -[(x - \frac{3}{2})^2] + \frac{41}{4}$ opens downward and has vertex $(\frac{3}{2}, \frac{41}{4})$. So we shift the graph of $y = x^2$ right by $\frac{3}{2}$, move it down by $\frac{41}{4}$ and then reflect it across the x-axis. (Or we shift the graph of $y = x^2$ right by $\frac{3}{2}$, reflect it across the y-axis and then move it *up* by $\frac{41}{4}$.)

(e) From the previous problem we see that graph of $y = 2(x - 2)^2 + 1$ has vertex $(2, 1)$. So we do the following operations, *in this order*:

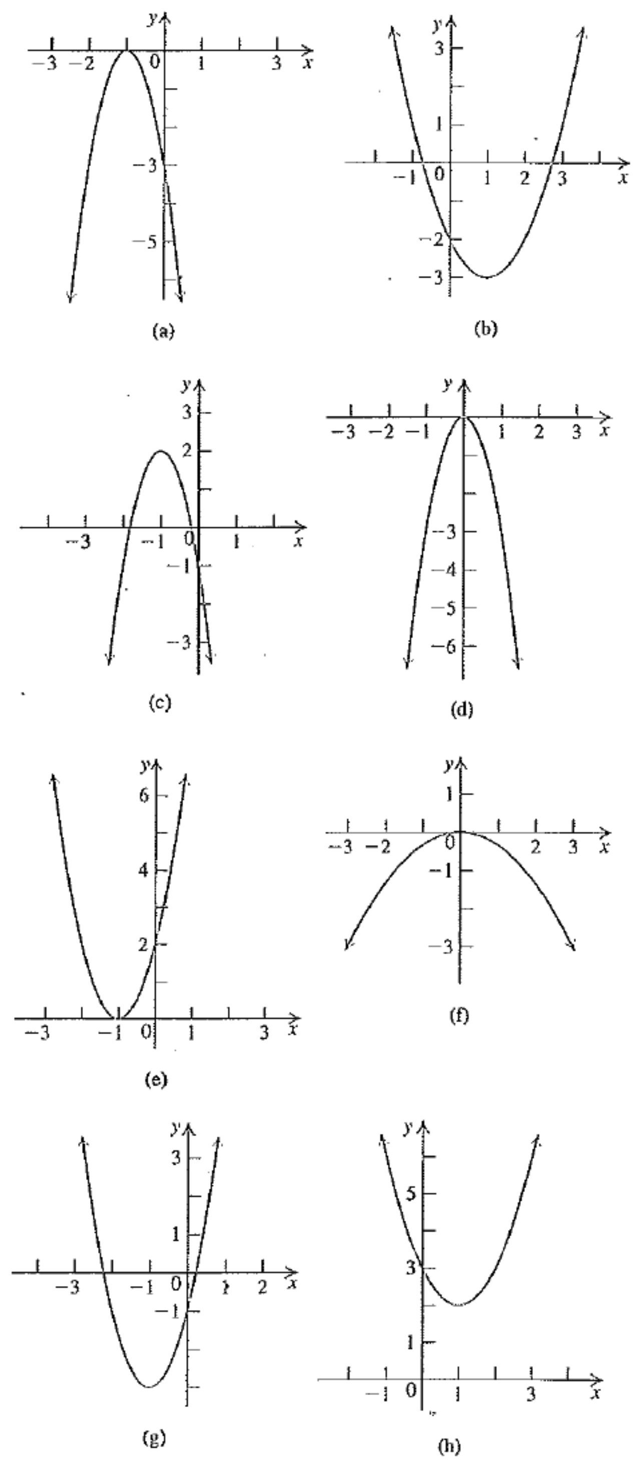
i. first shift the graph of $y = x^2$ right by 2,

ii. then stretch the graph vertically by a factor of 2,

iii. and then shift the graph up by 1.

(Worksheet 2.1A is continued on the next page.)

4. Find the equation for the parabolas below. Put your answers in standard form.



Solutions.

(a) Since the parabola has x-intercept at $x = -1$, with multiplicity 2, then it must be of the form $y = a(x+1)^2$. Since it goes through $(0, -3)$ the equation is $\boxed{y = -3(x+1)^2}$.

(b) The zeroes seem to be about 2.7 and -0.7 with vertex at $(1, -3)$. The standard form for this parabola must be something like $f(x) = a(x-1)^2 - 3$. In this case, $a = 1$ works perfectly well so the equation is $\boxed{y = (x-1)^2 - 3.}$

(c) The vertex is at $(-1, 2)$ so the quadratic has form $f(x) = a(x+1)^2 + 2$. Since the parabola goes through $(0, -1)$, a must be -3. So the equation is $\boxed{y = -3(x+1)^2 + 2.}$

(d) The vertex is at $(0, 0)$ so the quadratic has form $f(x) = ax^2$. It appears that a is about -3 since the parabola probably goes through $(1, -3)$. So the equation is $\boxed{y = -3x^2.}$

(e) The vertex is at $(-1, 0)$ so the quadratic has form $f(x) = a(x+1)^2$. Since the parabola goes through $(0, 2)$, a must be equal to 2. So the equation is $\boxed{y = 2(x+1)^2.}$

(f) The vertex is at $(0, 0)$ so the quadratic has form $f(x) = ax^2$. It appears that a is about $-\dfrac{1}{4}$ since the parabola probably goes through $(2, -1)$. So the equation is $\boxed{y = -\dfrac{1}{4}x^2.}$

(g) The vertex is at $(-1, -3)$ so the quadratic has form $f(x) = a(x+1)^2 - 3$. Since the parabola goes through $(0, -1)$, $a = 2$ and so the equation is $\boxed{y = 2(x+1)^2 - 3.}$

(h) The vertex is at $(1, -2)$ so the quadratic has form $f(x) = a(x-1)^2 + 2$. Since the parabola goes through $(0, 3)$, $a = 1$ and so the equation is $\boxed{y = (x-1)^2 + 2.}$