More exercises on slope

We continue to study the slope of a line, working through several examples.

Find the equation for the line passing through the points \((3, 8)\) and \((6, 20)\). Put your answer in point-slope form.

**Solution.** First we find the slope of the line passing through \((3, 8)\) and \((6, 20)\). It is \(\frac{20 - 8}{6 - 3} = \frac{12}{3} = 4\).

Since the line goes through the point \((6, 20)\) and has slope 4 then the slope is

\[ 4 = \frac{y - 20}{x - 6}. \]

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A related problem Find the $x$-intercept of the line passing through the points $(3, 8)$ and $(6, 20)$.

**Solution.** Set $y = 0$ in the solution from the previous problem:

$$4(x - 6) = 0 - 20$$

$$(x - 6) = \frac{-20}{4}$$

$$x - 6 = -5$$

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The $x$-intercept is $(1, 0)$. 
Working some examples on slope

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The $x$-intercept is $(1, 0)$. 
The line \( y = f(x) \) has slope 4 and passes through the point \((12400, 999900)\). Find \( f(12402) \).

Let's keep this as simple as possible.

**Solution.** A line of slope 4 has the property that every step to the right creates a rise of 4.

So 2 steps to the right (from \( x = 12400 \) to \( x = 12402 \)) creates a rise of 8.

Since we began at height 999900 we must end at \( 999900 + 8 = 999908 \).

Notice that we did not need to deal with any equations. Merely understanding the *meaning* of slope was enough to quickly do this problem!
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The graph of $y = f(x)$ is a straight line of slope 12.

If $f(gazillion) = google$ then what is $f(gazillion + 3)$?

**Solution.** $f(gazillion + 3) = google + (3)(12) = google + 36$.

That's it!
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Easy!
The average rate of change of a function

Suppose two points \( P \) and \( Q \) are on the graph \( y = f(x) \) of a function \( f \).

Since \( f \) is a function, then by the vertical line test, these two points cannot have the same \( x \)-value.

Concentrate on the point \( P \) and write its coordinates as \((x, f(x))\).

The other point, \( Q \) has \( x \)-coordinate \( x + h \) for some value of \( h \).
(\( h \) is the “run” between the points \( P \) and \( Q \).)

The coordinates of \( Q \) are \( Q(x + h, f(x + h)) \).

The slope of the line joining \( P \) to \( Q \) is

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m := \frac{\Delta y}{\Delta x} = \frac{f(x + h) - f(x)}{(x + h) - x} = \frac{f(x + h) - f(x)}{h}. \tag{5}
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This expression, \( \frac{f(x+h)-f(x)}{h} \), is the difference quotient, the slope of the line connecting the points \( P(x, f(x)) \) and \( Q(x + h, f(x + h)) \).

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**Worked Examples.** Consider the quadratic function \( f(x) = x^2 \). Find the difference quotient for this function.

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