**Worksheet 1.4A, Symmetries of functions**
MATH 1410
(SOLUTIONS)

1. Graph the functions below and decide if they are even, odd, or neither even nor odd.

   (a) $f(x) = 3x^4 + 3$

   (b) $f(x) = 2x^3 - x$

   (c) $f(x) = 2x^3 - x + 2$

   (d) $f(x) = \dfrac{1}{x^2+1}$

   (e) $f(x) = \dfrac{x}{x^2+1}$

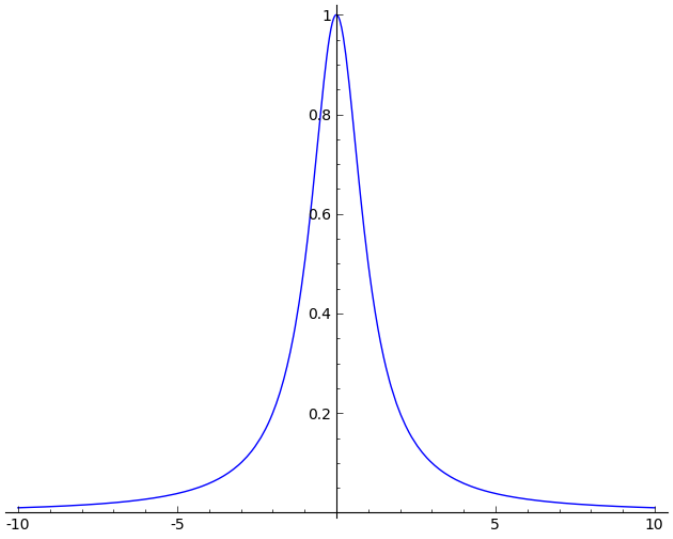
**Solutions.**

(a) $f(x) = 3x^4 + 3$ is even.

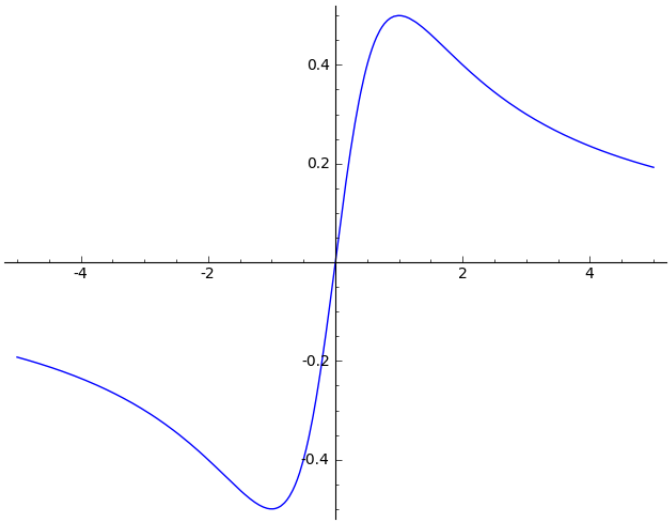
(b) $f(x) = 2x^3 - x$ is odd.

(c) $f(x) = 2x^3 - x + 2$ is neither even nor odd.

(d) $f(x) = \dfrac{1}{x^2+1}$ is even. Here is the graph:

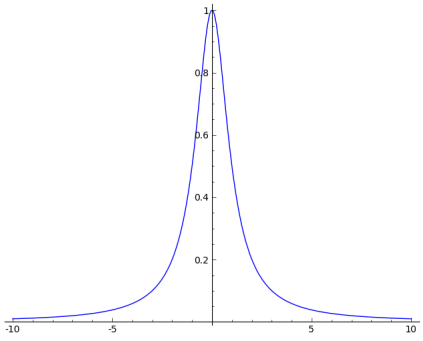



(e) $f(x) = \dfrac{x}{x^2+1}$ is odd. Here is the graph:

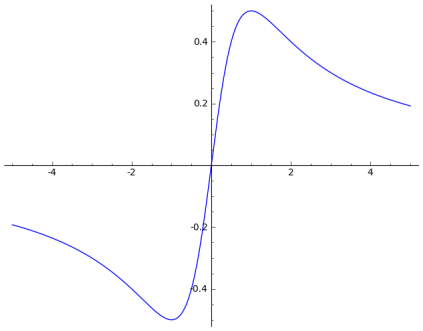



2. You are given the graphs of certain functions. Determine if the function is even, odd, or neither.

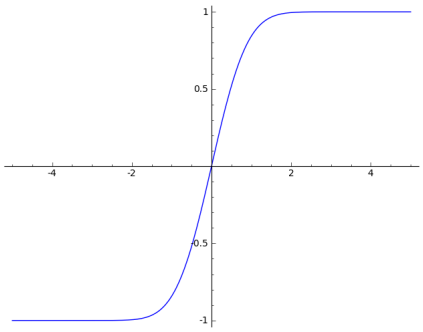
(a)



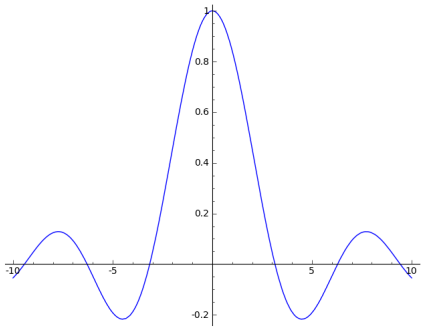

(b)




(c)




(d)




**Solutions.**

(a) Even

(b) Odd

(c) Odd

(d) Even

3. Decide algebraically if the function is even, odd, or neither.

   (a) $f(x) = x^3 - 4x$

   (b) $f(x) = \dfrac{x}{1+x^2}$

   (c) $f(x) = x^5 + 7x^2 - 3x + 5$

   **Solution.**

   (a) The function $f(x) = x^3 - 4x$ is odd since it is symmetric about the origin. We can check this algebraically:

   $$f(-x) = (-x)^3 - 4(-x) = -x^3 + 4 = -(x^3 - 4) = -f(x).$$

   (This is done in the lecture notes.)

   (b) If $f(x) = \dfrac{x}{1+x^2}$ then $f(-x) = \dfrac{-x}{1+(-x)^2}$. Since $(-x)^2 = x^2$ we can simplify this to

   $$f(-x) = \frac{-x}{1+(-x)^2} = -\frac{x}{1+x^2} = -f(x).$$

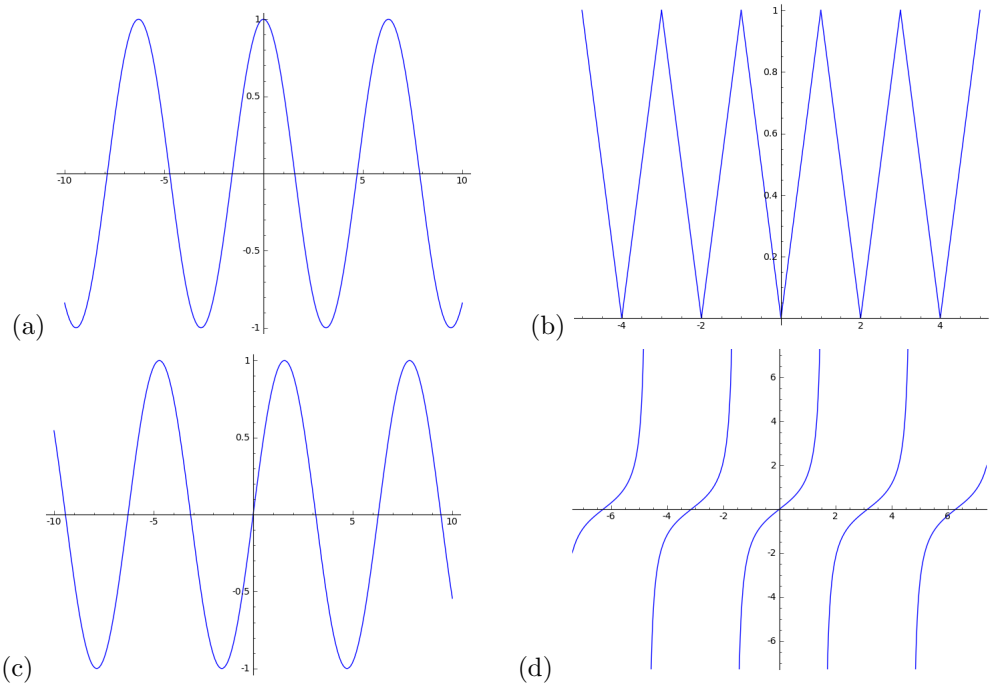
   So $f(x)$ is odd. (This is done in the lecture notes.)

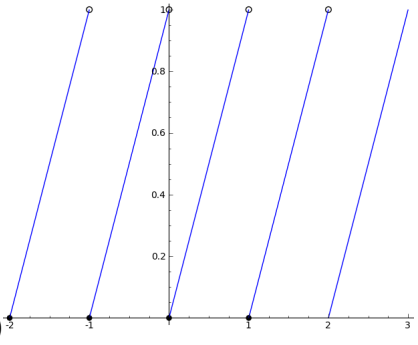
   (c) If $f(x) = x^5 + 7x^2 - 3x + 5$ then

   $$f(-x) = (-x)^5 + 7(-x)^2 - 3(-x) + 5 = -x^5 + 7x^2 + 3x + 5.$$

   Since $f(-x) = -x^5 + 7x^2 + 3x + 5$ is neither equal to $f(x)$ nor equal to $-f(x)$ then $f(x)$ is neither even nor odd. (This is done in the lecture notes.)

4. Look carefully at the graphs of the following periodic functions and estimate their period.



(e) 


**Solutions.**

(a) The period is $2\pi$, slightly more than 6.

(b) The period is 2.

(c) The period is $2\pi$, slightly more than 6.

(d) The period is $\pi$, slightly more than 3.

(e) The period is 2.

5. There is a function which is both even and odd! What is it?

**Solution.** If $f(x) = 0$ then the graph of $y = 0$ is just the $x$-axis. This has both odd and even symmetry!