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![Diagram of a function machine](image)

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We give an example (from Wikipedia) of a function from a set $X$ to the set $Y$.

The function maps 1 to $D$, 2 to $C$ and 3 to $C$.

Note that each element of $X$ has a unique output in $Y$. 
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However the map below is not a function.

Some items in $X$ are not mapped anywhere; worse, the item 2 has two outputs, both $B$ and $C$.

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Functions as questions

Functions occur naturally in our world.

When we pull out an attribute of an object, we are essentially creating a function.

For example, the set $X$ below has polygons with various colors. The question, “What is the color of a polygon?” could be viewed as a function that maps to polygons to colors.

\[ X \quad \rightarrow \quad Y \]

\[ \triangle \rightarrow \text{red} \]
\[ \square \rightarrow \text{yellow} \]
\[ \square \rightarrow \text{green} \]
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Functions occur throughout our modern technological society.

The US social security number is a function $SSN$ mapping US citizens to nine digit numbers.

At Sam Houston State University, all students and staff are assigned a Sam ID. This as a function $SamID$, mapping students/staff to nine digit numbers.

For example,

$$SamID(Ken \ W \ Smith) = 000354765.$$

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We may sometimes define a function by a table or by a list of ordered pairs.

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A worked exercise

**Worked Exercise.** Consider the function with domain $D = \{-2, -1, 0, 1, 2\}$, codomain the real numbers $\mathbb{R}$, defined by the formula $g(x) = x^2$.

1. Display the function $g$ in tabular form, and
2. Display the function $g$ as a set of ordered pairs.
3. Give the range of the function $g$.

**Solution.**

1. As a table, we can write out the function $g$ as

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<tr>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
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<td>2</td>
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2. As a set of order pairs, $g = \{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$
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   $\begin{array}{|c|c|}
   \hline
   x & g(x) \\
   \hline
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   -1 & 1 \\
   0 & 0 \\
   1 & 1 \\
   2 & 4 \\
   \hline
   \end{array}$

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**Worked Exercise.** Consider the function with domain $D = \{-2, -1, 0, 1, 2\}$, codomain the real numbers $\mathbb{R}$, defined by the formula $g(x) = x^2$.

1. Display the function $g$ in tabular form, and
2. Display the function $g$ as a set of ordered pairs.
3. Give the range of the function $g$.

**Solution.**

1. As a table, we can write out the function $g$ as

<table>
<thead>
<tr>
<th>$x$</th>
<th>$g(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
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2. As a set of order pairs, $g = \{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$

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Another example. Consider the function defined earlier.

Write this function in both tabular form and as a set of ordered pairs.

Solution. In tabular form we have:

<table>
<thead>
<tr>
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<tbody>
<tr>
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<td>D</td>
</tr>
<tr>
<td>2</td>
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<tr>
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![Diagram](image)

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![Diagram of a function with inputs 1, 2, 3 mapping to outputs D, B, C]

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Definition of a function

In the next lecture we examine functions defined by equations.
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(END)