Symmetric Connectivity in WSNs Equipped with Multiple Directional Antennas

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Abstract—This paper concerns wireless sensor networks (WSNs) of which each node is equipped with \( k \geq 1 \) directional antennas having beam-width \( \theta \in (0, 2\pi) \). The sum \( \varphi_k \) of the beam-widths of the \( k \) antennas of each node is in \((0, 2\pi)\). Each node is initially assigned a transmission range \( r = O(1) \) such that the induced symmetric communication graph (SCG) is connected.

In this paper, we study the AO problem assuming that each node has two antennas \((k = 2)\) each of which has beam-width \( \theta = \pi/3 \) or \( \pi/4 \). We propose two approximation algorithms that orient the antennas to yield symmetric connected communication graphs (SCCGs) where the transmission power ranges are bounded by 4 and 5 when \( \theta = \pi/3 \) and \( \pi/4 \), respectively. These bounds are the first results for this problem. We also study the performance of our algorithms through simulation.

I. INTRODUCTION

One of the main issues concerning WSNs is energy consumption because tiny sensor devices have limited battery. In order to conserve energy of sensor nodes, topology control has been widely used in omnidirectional WSNs [1]–[9]. By assigning appropriate power ranges to the nodes, an energy efficient network with desired features, such as connectivity, low interference and fault tolerance [10] can be established.

Recently, topology control has been investigated in directional WSNs as well. An interesting problem concerning topology control is to establish connectivity, which is known as the Antenna Orientation (AO) problem, where we are given a WSN in which each node is equipped with \( k \geq 1 \) directional antennas having beam-width \( \theta \in (0, 2\pi) \). The sum \( \varphi_k \) of the beam-widths of the \( k \) antennas of each node is in \((0, 2\pi)\), and each node is initially assigned a transmission range 1 that yields a connected unit disk graph spanning all nodes. The objective of the AO problem concerning symmetric connectivity is to compute an orientation of the antennas and to find a minimum transmission power range \( r = O(1) \) such that the induced symmetric communication graph (SCG) is connected.

When \( k = 1 \), concerning symmetric connectivity, Carmi et al [11] showed that if \( \theta \geq \pi/3 \) and \( r \) is unbounded, it is always possible to orient the antennas to induce a SCCG. Aschner et al [12] later showed that when \( |S| = 4 \) and \( \theta = \pi/2 \), it is possible to orient antennas to induce a SCCG, and the union of the corresponding broadcasting sectors covers the entire plane. They also presented an algorithm that yields a SCCG where the assigned range is \( 14\sqrt{2} \). Later, Aschner et al [13] proposed another algorithm yielding a SCCG where the assigned range is 7 when \( \theta = 2\pi/3 \). Recently, Tran et al [14] introduced an algorithm yielding a SCCG where the assigned range is improved to 6 when \( \theta = 2\pi/3 \). The same authors [15] proposed another algorithm for \( \theta = \pi/2 \) where the range is bounded by 9 which is a significant improvement over the bound of \( 14\sqrt{2} \) obtained by Aschner et al [12].

TABLE I: Results of the antenna orientation problem concerning strong connectivity when \( 2 \leq k \leq 5 \) ([16], [17]).

<table>
<thead>
<tr>
<th>( k )</th>
<th>( \varphi_k )</th>
<th>assigned range</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>( 2\pi/3 \leq \varphi_2 )</td>
<td>( 2\pi/3 \leq \varphi_2 )</td>
</tr>
<tr>
<td>3</td>
<td>( 0 \leq \varphi_3 )</td>
<td>( \sqrt{3} )</td>
</tr>
<tr>
<td>4</td>
<td>( 0 \leq \varphi_4 )</td>
<td>( \sqrt{3} )</td>
</tr>
<tr>
<td>5</td>
<td>( \varphi_5 \geq 0 )</td>
<td>1</td>
</tr>
</tbody>
</table>

In this paper, we continue the study of the Antenna Orientation (AO) problem concerning symmetric connectivity assuming that each node has two antennas (i.e., \( k = 2 \)). The objective is to compute orientations of antennas with angle \( \theta = \pi/3 \) or \( \theta = \pi/4 \) and to find a minimum transmission power range \( r = O(1) \) such that the induced SCG is connected. For \( \theta < 2\pi/3 \), the AO problem can be shown to be NP-hard by a simple reduction from the Hamilton cycle problem in hexagonal grid graphs, which is known to be NP-complete.
Let $G$ will independently orient the antennas of nodes in $F$ into groups based on an MST, and the Antenna Orientation Algorithm that independently orients the antennas for each group. We also derive a bound for the required power range.

1) Nodes partition algorithm: The algorithm starts by constructing a Euclidean MST $T$ spanning the set $S$ of nodes. Let $T$ be rooted at an arbitrary node $s \in S$. We denote the parent of a node $v$ by $v.p$ and a child of $v$ by $v.c$. Let $V.C = \{v.c_1, v.c_2, \ldots, v.c_k\}$ be the set of children of node $v$ in $T$. Next, we partition the nodes in tree $T$ into a collection $G$ of 3-node groups and a set $B$ of bridge nodes.

Let $F$ be the forest that initially includes tree $T$. The Partition Algorithm starts by picking an arbitrary tree in forest $F$ with root $s$. If $|s.C| \leq 1$, we add $s$ to $B$. (Nodes in $B$ will use their antennas to connect its only child to its parent node in $T$.) If $|s.C| > 1$, we arbitrarily choose two of its children and form a group of 3 nodes $G_i$ and add it to set $G$. We then remove $s$ and its group (if any) from $F$, and mark all node(s) adjacent to $s$ or its group (if any) as root(s) of new tree(s) in $F$. The process is repeated until $F = \phi$. The details are shown in Algorithm 1.

2) Antennas orientation algorithm: In this subsection, we will independently orient the antennas of nodes in $G$ and $B$ to create a SCCG for $S$.

### Algorithm 1

#### Partition algorithm for WSNs in which each node is equipped with two $\pi/3$ directional antennas

**Input:** A set $S$ of nodes

**Output:** A collection $G$ of 3-node groups and a set $B$ of bridge nodes.

1: Construct a Euclidean MST $T$ spanning $S$ with root $s$
2: Let $F$ be the forest containing $T$
3: Let $G$ and $B$ be empty initially.
4: repeat
5: Pick a tree $T_i$ in $F$ and let $s$ be its root.
6: if $|s.C| \leq 1$ then
7: Add $s$ to $B$ and remove $s$ from $F$
8: else
9: Form group $G_i$ from $s$ and two of its children
10: Add $G_i$ to $G$ and remove $G_i$ from $F$
11: end if
12: Mark all nodes that are adjacent to $s$ or its group (if any) as root(s) of new tree(s) in $F$
13: until $|F| = 0$
14: return $G$ and $B$

**Connectivity for each group in $G$:** Assume that every node is assigned a sufficient range so that it can connect to any node in its broadcasting sectors. We will orient antennas of each group in $G$ independently. For each node in group $G_i$ of $G$, we join both of its $\pi/3$ antenna sectors together to form a $2\pi/3$ sector. Then we apply an orientation of single $2\pi/3$ directional antennas by Ashner et al in Lemma 2.1 [13]. The result can be stated as follow:

**Lemma 1.** Let $S = \{a, b, c\}$ be a set of three nodes in the plane, and each node has two directional antennas with beam-width $\theta = \pi/3$ and range $= \infty$. Then, one can orient the antennas of $S$, such that the induced SCCG of $S$ is connected and the broadcasting sectors of $S$ cover the entire plane.

**Symmetric connectivity between groups in $G$:** As the antennas in each group are independently oriented, we can make use of another result by Aschner in [13], again by combining 2 antennas into one, to obtain symmetric connectivity between two adjacent groups.

**Lemma 2.** (Theorem 2.4 in [13]) Let $S_1 = \{a, b, c\}$ and $S_2 = \{x, y, z\}$ be two sets of three nodes each, where every node is equipped with two $\pi/3$ directional antennas and range $= \infty$. Assume that the broadcasting sectors of $S_1$ and, independently, of $S_2$ are oriented according to Lemma 1. Then, the induced graph $G_{S_1 \cup S_2}$ is symmetric connected.

For a bridge node $v$ in $B$, we will use its two antennas to establish connectivity between its parent and its only child (if any). If both its parent and its child are in $B$, we simply use each of its antennas to connect to its parent and its child, respectively, as shown in Fig. 1(a). Otherwise, either its parent or its child is in a 3-node group. In this case, $v$ is connected to such groups by directing its antennas to the nodes that cover $v$. (See Fig. 1(b)).
The idea of our Orientation Algorithm is illustrated in Fig. 2 and the details are shown in Algorithm 2.

![Diagram of bridge nodes](image)

**Fig. 1: Connectivity of bridge nodes**

**Algorithm 2** Antennas Orientation Algorithm for directional WSNs where \( k = 2 \) and \( \theta = \pi/3 \)

**Input:** A collection \( G \) of 3-node groups and a set \( B \) of bridge nodes

**Output:** An orientation of antennas of nodes in \( S \)

1. **Orientation of nodes in \( G \)**
2. **repeat**
3. Pick a group \( G_i \) in \( G \)
4. Orient antennas of nodes in \( G_i \) according to Lemma 1
5. Remove \( G_i \) from \( G \)
6. **until** \( |G| = 0 \)
7. **Orientation of nodes in \( B \)**
8. **repeat**
9. Pick node \( v \) in \( B \)
10. let \( U = \{u_1, u_2\} \) be parent and child of \( v \) in tree \( T \)
11. **for all** \( u_i \) in \( U \) **do**
12. if \( u_i \) has at most one child **then**
13. Direct one of \( v \)'s antennas to \( u_i \)
14. **else**
15. let \( G_i \) be the group that contains \( u_i \)
16. let \( t \) be the node in \( G_i \) that covers \( v \)
17. Direct one of \( v \)'s antennas to \( t \)
18. **end if**
19. **end for**
20. Remove \( v \) from \( B \)
21. **until** \( B = \phi \)

**IV. Symmetric Connectivity in Directional WSNs Where \( k = 2 \) and \( \theta = \pi/4 \)**

In this section, we apply the idea in the previous section to directional WSNs where \( k = 2 \) and \( \theta = \pi/4 \). By joining both antenna sectors together, we can form a single directional antenna with beam-width \( \pi/2 \) at each node. We then apply a construction by Aschner et al in Theorem 2.1 in [12] to form a 4-node structure that can cover the entire plane as stated in the following lemma.

**Lemma 4.** Let \( S \) be a set of 4 nodes where each node is equipped with two directional antennas with beam-width \( \theta = \pi/4 \) and range \( r = \infty \). Then, one can orient the antennas of nodes in \( S \) such that the induced SCG of \( S \) is connected and the broadcasting sectors of \( S \) cover the entire plane.
A. Partition algorithm

Our partition algorithm partitions our network into bridge nodes, full groups and partial groups. A full group consists of 4 nodes and can orient their antennas to cover all the nodes around it. Unlike a full group, a partial group has 3 nodes and can create at most 3 symmetric connections with other nodes. The details are shown in Algorithm 3:

Algorithm 3 Partition algorithm for directional WSNs where \( k = 2 \) and \( \theta = \pi/4 \)

Input: A set \( S \) of nodes

Output: A set of full groups \( G_F \), a set of partial groups \( G_P \) and a set of bridge nodes \( B \).

1: Construct a Euclidean MST \( T \) spanning \( S \) with root \( s \)
2: Let \( F \) be a forest containing \( T \)
3: Let \( G_F, G_P \) and \( B \) be empty initially
4: repeat
5:   Pick a tree \( T_i \) in \( F \) with root \( s \)
6:   if \( |s.C| \leq 1 \) then
7:     Add \( s \) into set \( B \)
8:   else
9:     if \( |s.C| \geq 3 \) or \( \exists s.c_i : |s.c_i.C| \geq 2 \) then
10:        Form a full group that consists of \( s \) and:
11:           - 3 of \( s \)'s children (Fig. 4(a))
12:           - OR \( s.c_i \) and \( s.c_j \) two children (Fig. 4(b))
13:        Add the group to set \( G_F \)
14:   else
15:        Form a partial group that consists of \( s \) and its two children (Fig. 4(c));
16:        Add the group to set \( G_P \)
17:   end if
18: end if
19: Remove \( s \) and its group (if any) from \( F \)
20: Mark the nodes adjacent to \( s \) or its group (if any) as root(s) of new tree(s) in \( F \)
21: until \( |F| = 0 \)
22: return \( G_F, G_P \) and \( B \)

B. Orientation algorithm

In the following we describe our orientation algorithm for nodes in sets \( G_F, G_P \) and \( B \). We then prove that we can always form symmetric connections between any node and its adjacent nodes. First, consider two neighboring full groups:

Lemma 5. Let \( G_1 \) and \( G_2 \) be full groups (of 4 nodes each) in which each node is equipped with a pair of \( \pi/4 \) directional antennas. By orienting antennas of each group independently as in Lemma 4 and assigning a sufficient range, the induced SCG of \( G_1 \cup G_2 \) is connected.

Proof. First, notice that a full group as depicted in Fig. 4(a)(b) is always formed by 3 edges of MST \( T \) which are incident to a single node. We now prove the lemma by the following facts:

Fact 1. Let \((a, b)\) be an edge in MST \( T \). By the properties of Euclidean MSTs, there is no other node of \( T \) that resides in the circle with diameter \((a, b)\). (See Fig. 5(a)).

Fig. 5: (i) No node in circle with diameter \((a, b)\), (ii) No node can reside inside a full group

Fact 2. Since a full group is formed by MST edges, no node can reside inside a full group as depicted in Fig. 5(b).

Fact 3. Let \( G_1 \) and \( G_2 \) be full groups constructed by Algorithm 3. Since \( G_1 \) and \( G_2 \) are formed by Delaunay triangles, they cannot intersect each other. Furthermore, \( G_1 \) does not reside inside \( G_2 \) and vice versa. Therefore, \( G_1 \) and \( G_2 \) are separated by a line.

Now consider 2 full groups \( G_1 \) and \( G_2 \) where each group is oriented as done in Lemma 4. As they are separated by a line, according to a result from Aschner [12], Theorem 3.1, the induced SCG of \( G_1 \cup G_2 \) is connected. \( \square \)

To deal with partial groups we prove the following lemma.

Lemma 6. Let \( G_i = \{s, a, b\} \) be a partial group rooted at \( s \) where each node is equipped with two \( \pi/4 \) directional antennas as depicted in 4(c). Let \( p \), parent of \( s \), be the parent of \( G_i \) and let the children of \( a \) and \( b \) be the children of \( G_i \). Then we can orient the antennas in \( G_i \) to create symmetric connections between nodes in \( G_i \), its parent and its children.

Proof. First, note that if node \( p \) is in a full group, then given a sufficient range, the antennas in \( p \)'s group can cover nodes \( s, a \) and \( b \). We can connect \( s, a \) and \( b \) to \( p \)'s group by one antenna and use the other antenna of \( a \) and \( b \) to connect to their children.

Otherwise, \( p \) does not belong to any full group. Since each of \( a \) and \( b \) has at most 1 child, group \( G_i \) has at most 2 children. As \( s \) connects to \( a \) and \( b \) through 2 edges of \( T \), there are 2 possible cases:

- \( s \) is inside triangle \( \triangle pab \) as depicted in Fig. 6(a): In this case, \( p \) uses one antenna to connect to its parent.
Nodes $a$ and $b$ use one antenna each to connect to their respective child. Hence, to form symmetric connections between $s, p, a$ and $b$, $s$ uses both of its antennas and each of $p, a$ and $b$ uses at most one antenna per node. Consider the 6 angles created by $s$ and any two of $p, a, b$ whose vertices are $p, a, b$ as depicted in Fig. 6. Since the sum of these angles is $\pi$, there must be at least 2 angles that are $< \pi/4$. Without loss of generality, assume that $\angle spa$ is $< \pi/4$. Then we can form symmetric connections for $s, a, b$ and $p$ as depicted in Fig. 6(b). Fig. 6(c) shows how to form symmetric connections if the vertex of the angle is either $a$ or $b$.

- $s$ is outside triangle $\triangle pac$ as depicted in Fig. 7(a): In this case, because $\angle psa$ is at least $2\pi/3$ (due to properties of Euclidean MSTs), either $\angle sap$ or $\angle spa$ is $< \pi/4$. In either case, we can orient the antennas to form symmetric connections as shown in Fig. 7, (b) and (c).

Thus, we can orient the antennas in $G_i$ to form symmetric connections between nodes in $G_i$, its parent and its children. For convenience let’s call the node(s) in $G_i$ covered by one of $p$’s antennas the corresponding node(s) of $p$ in $G_i$.

**Algorithm 4** Antennas Orientation Algorithm for directional WSNs where $k = 2$ and $\theta = \pi/4$

**Input:** Sets $G_T$, $G_P$ and $B$

**Output:** Antenna orientation of nodes in network $S$

```python
Orientation of nodes in $G_T$
1: repeat
2: Pick a group $G_i$ in $G_T$
3: Orient antennas of nodes in $G_i$ as in Lemma 4
4: Remove $G_i$ from $G_T$
5: until $G_T = \phi$

Orientation of nodes in $G_P$
6: repeat
7: Pick a group $G_i$ in $G_P$
8: Orient antennas of nodes in $G_i$ as in Lemma 6
9: Remove $G_i$ from $G_P$
10: until $G_P = \phi$

Orientation of nodes in $B$
11: repeat
12: Pick node $v$ in $B$
13: let $U = \{u_1, u_2\}$ be set of adjacent nodes of $v$
14: for all $u_i$ in $U$ do
15: if $u_i$ is in a full group then
16: let $G_i$ be the group that contains $u_i$
17: let $t$ be the node in $G_i$ that covers $v$
18: Direct one of $v$’s antennas to $t$
19: else
20: if $u_i$ is the root of a partial group $G_k$ then
21: Direct one of $v$’s antennas to the corresponding node(s) of $G_k$
22: else
23: Direct one of $v$’s antennas to $u_i$
24: end if
25: end if
26: end for
27: Remove $v$ from $B$
28: until $B = \phi$
```

**Theorem 7.** Given a set of nodes $S$ in the plane and $UDG(S)$, where each node is equipped with a pair of $\pi/4$ directional antennas and range $r = 5$, one can orient the antennas of nodes in $S$ such that the induced SCCG is connected.

**Proof.** We apply Algorithm 3 to partition $S$ and then Algorithm 4 to orient the antennas of nodes in $S$. We now show that range $r = 5$ is sufficient to obtain a SCCG for $S$. Consider two adjacent nodes $p$ and $c$ in MST $T$ that can be in a full group, in a partial group or is a bridge node. Since the range required to connect two bridge nodes together or a bridge node to another node in its adjacent group is small compared to the range required to build the connections between full groups or partial groups, let’s consider the cases where $p$ and $c$ are in full group(s) or partial group(s).

- $p$ and $c$ are in the same group: If $p$ and $c$ are in a full group as in Fig. 4(a),(b), then, obviously the distance between $p$ and $c$ is at most 2 hops. If $p$ and $c$ are in a partial group as in Fig. 4(c), then the distance between $p$ and $c$ is 1 hop.
- $p$ and $c$ are in different groups:
  - $p$ and $c$ are both in partial groups as in Fig. 8(a): $p$ uses one antenna to connect to its corresponding node(s) in $c$’s group to form a symmetric connection. The maximum distance between $p$ and that node in $c$’s group is 2 hops.
  - $p$ is in a partial group and $c$ is in a full group as in Fig. 8(b): Since nodes in $c$’s group can cover all other
nodes around it, to form a symmetric connection between \( c \) and \( p \), \( p \) connects to the node in \( c \)’s group that covers it. Their maximum distance is 3 hops.

- \( p \) is in a full group and \( c \) is in a partial group as in Fig. 8(c): Similar to the previous case, \( c \) uses one antenna to connect to the node that covers it in \( p \)'s group which is at most 3 hops away.
- \( p \) and \( c \) are in full groups as in Fig. 8(d): In this case, we need a connection between 2 full groups to connect \( c \) and \( p \) together. Obviously, the maximum distance between 2 nodes in two adjacent full groups is 5 hops.

Thus, range \( r = 5 \) is sufficient to establish a SCCG for \( S \).

![Fig. 8: Connections between parent nodes and child nodes](image)

V. SIMULATION

In our simulation, we randomly deploy nodes on an area of dimension \( 10000 \times 10000 \), and then choose a uniform range such that the induced UDG results in a dense network, a sparse network or a medium network. If number of edges in the UDG is \( \geq n^2/4 \) (\( \leq 4n \)), it is said to be dense (sparse); otherwise it is a medium network. We then apply our method on each type of networks and compare the results against our previous methods in [15] and [14] which currently give best range assignment results for WSNs equipped with single directional antennas.

As we can see in Tables II and III, although total beam-width is the same for both cases, our simulation shows that using 2 directional antennas is more effective than using just one as far as power range is concerned. Note that in some cases \( r \) is \(< 1 \). This is especially true for dense networks when the unit range is large to create sufficient edges in the UDGs.

**TABLE II: Uniform range for DWSNs with \( n \) nodes where total beam-width is \( \pi/2 \)**

<table>
<thead>
<tr>
<th>( n )</th>
<th>Sparse network (( k = 1 ))</th>
<th>Medium network (( k = 1 ))</th>
<th>Dense network (( k = 2 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>2.079</td>
<td>1.55</td>
<td>0.969</td>
</tr>
<tr>
<td>100</td>
<td>2.181</td>
<td>1.684</td>
<td>0.832</td>
</tr>
<tr>
<td>250</td>
<td>2.257</td>
<td>1.793</td>
<td>0.801</td>
</tr>
</tbody>
</table>

**TABLE III: Uniform range for DWSNs with \( n \) nodes where total beam-width is \( 2\pi/3 \)**

<table>
<thead>
<tr>
<th>( n )</th>
<th>Sparse network (( k = 1 ))</th>
<th>Medium network (( k = 2 ))</th>
<th>Dense network (( k = 5 ))</th>
</tr>
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<tbody>
<tr>
<td>50</td>
<td>1.761</td>
<td>1.497</td>
<td>0.508</td>
</tr>
<tr>
<td>100</td>
<td>1.846</td>
<td>1.583</td>
<td>0.926</td>
</tr>
<tr>
<td>250</td>
<td>1.911</td>
<td>1.663</td>
<td>0.67</td>
</tr>
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</table>

obtained by our algorithms are 4 and 5. Through simulations we also show that, in term of power range, using 2 directional antennas is better than one. As to future work, we will focus on how to improve these bounds and extend our method for directional antennas with smaller beam-widths.

REFERENCES


VI. CONCLUSION

In this paper, we studied the Antenna Orientation (AO) problem for DWSNs in which each node is equipped with two directional antennas, each having beam-width \( \pi/4 \) or \( \pi/3 \). As the problems are NP-hard, we proposed two constant-factor approximation algorithms. We proved that the power range