1. Evaluate the function $f$ at the prescribed values and then graph the function.

(a) $f(x) = \begin{cases} 2x + 1, & \text{if } x \leq 0 \\ x^2 + 1, & \text{if } x > 0 \end{cases}$

Compute $f(2)$, $f(0)$, $f(-2)$ and graph this function.

**Solutions.**

Here, if $x \leq 0$ then the function obeys the rule $f(x) = 2x + 1$. Thus $f(-2) = 2(-2) + 1 = -3$ and $f(0) = 2(0) + 1 = 1$. But if $x > 0$ then $f(x) = x^2 + 1$ and so $f(2) = 2^2 + 1 = 5$.

The graph of the function appears in figure 6. Notice how the two “pieces” are glued together; we can see the line $y = 2x + 1$ in the region where $x \leq 0$ and the parabola $y = x^2 + 1$ in the region to the right of the $y$-axis.

(b) $f(x) = \begin{cases} 2x + 2, & \text{if } x \leq 1 \\ 4, & \text{if } x > 1 \end{cases}$

Compute $f(0)$, $f(1)$, $f(2)$ and $f(100)$. Then graph the function.

**Solution.** Since $x = 0$ and $x = 1$ both fall in the region where $x \leq 1$, we have that $f(0) = 2(0) + 2 = 2$ and $f(1) = 2(1) + 2 = 4$.

Since $x = 2$ and $x = 100$ both fall in the region where $x > 1$ then $f(2) = 4$ and $f(100) = 4$.

Here is the graph:
(c) $f(x) = \begin{cases} 
  x^2, & \text{if } x \leq 0 \\
  0, & \text{if } 0 < x \leq 1 \\
  2x - 2, & \text{if } x > 1
\end{cases}$

Compute $f(-1), f(0), f(1)$ and $f(2)$. Then graph the function.

**Solutions.**

Since $-1 \leq 0$ then $f(-1) = (-1)^2 = 1$.

Since $0 \leq 0$ then $f(0) = 0^2 = 0$.

Since $x = 1$ is in the interval $0 < x \leq 1$ then $f(1) = 0$.

Since $2 > 1$ then $f(2) = 2(2) - 2 = 2$.

(The graphs drawn in these examples were generated by the author using Sage.)

2. Graph the functions below and find the intervals where the function is (a) increasing, (b) decreasing:

(a) $f(x) = x^3 - x$. 
(b) $f(x) = x^3 - 4x$

(c) $f(x) = |x - 1|$

(d) $f(x) = \sqrt{x}$

Solutions.

(a) $f(x) = x^3 - x$ has graph

From the picture we can see that the function rises to around the point $(-0.6, 0.384)$, then drops to a point near $(0.6, -0.384)$ and then rises again after that. So, using these approximations, we would say that the graph increases in the (approximate) region $(-\infty, -0.6) \cup (0.6, \infty)$ while decreasing in the region $(-0.6, 0.6)$.

(Techniques from calculus, using the derivative, will eventually show that this function rises in the region $(-\infty, -\frac{1}{\sqrt{3}}) \cup (\frac{1}{\sqrt{3}}, \infty)$ and decreases in the region $(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$.)

(b) $f(x) = x^3 - 4x$ has graph

It rises in the region $(-\infty, -\frac{2}{\sqrt{3}}) \cup (\frac{2}{\sqrt{3}}, \infty)$. It drops in the interval $(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}})$. (Your answers should approximate these with decimals.)

(c) The graph of $f(x) = |x - 1|$ is
The graph rises in the interval $(1, \infty)$ and drops in the interval $(-\infty, 1)$.

(d) $f(x) = \sqrt[3]{x}$

The graph rises everywhere, that is, in the interval $(-\infty, \infty)$ and nowhere. (The region in which it is decreasing is the empty set $\emptyset$.

3. Find the $x$ and $y$ intercepts of the following functions:

(a) $f(x) = x^2 + 7x + 6$
(b) $f(x) = 2x^3 + 7x + 6$
(c) $f(x) = 2x^2 + 7x - 6$
(d) $f(x) = \frac{x^2 + 7x + 6}{x - 4}$
(e) $f(x) = \frac{x^2 + 7x + 6}{2x^2 + 7x + 6}$
Solutions.

(a) The $y$-intercept of $f(x) = x^2 + 7x + 6$ has $y$-coordinate $f(0) = 6$ so the $y$-intercept is $(0, 6)$. $x^2 + 7x + 6$ factors as $(x+1)(x+6)$ so the solutions to $x^2 + 7x + 6 = 0$ are $x = -1$ and $x = -6$. This means that the $x$-intercepts are $(-1, 0)$ and $(-6, 0)$.

(b) The $y$-intercept of $f(x) = 2x^2 + 7x + 6$ has $y$-coordinate $f(0) = 6$ so the $y$-intercept is $(0, 6)$. $2x^2 + 7x + 6$ factors as $(x+2)(2x+3)$ so the solutions to $2x^2 + 7x + 6 = 0$ are $x = -2$ and $x = -\frac{3}{2}$. This means that the $x$-intercepts are $(-2, 0)$ and $(-\frac{3}{2}, 0)$.

(c) The $y$-intercept of $f(x) = 2x^2 + 7x - 6$ has $y$-coordinate $f(0) = -6$ so the $y$-intercept is $(0, -6)$. The quadratic $2x^2 + 7x - 6$ does not have a nice factoring so we need to find the solutions to $2x^2 + 7x - 6$ by the quadratic formula. They are $x = \frac{-7 \pm \sqrt{97}}{4}$ so the $x$-intercepts are $(-\frac{7 + \sqrt{97}}{4}, 0)$ and $(-\frac{7 - \sqrt{97}}{4}, 0)$.

(d) The $y$-intercept of $f(x) = \frac{x^2 + 7x + 6}{x - 4}$ has $y$-coordinate $f(0) = \frac{6}{-4} = -\frac{3}{2}$ so the $y$-intercept is $(0, -\frac{3}{2})$.

The solutions to $f(x) = 0$ are the same as those for the function in part (a); the $x$-intercepts are $(-1, 0)$ and $(-6, 0)$.

(e) The $y$-intercept of $f(x) = 2x^2 + 7x + 6$ has $y$-coordinate $f(0) = \frac{6}{6} = 1$ so the $y$-intercept is $(0, 1)$.

The solutions to $f(x) = 0$ are the same as those for the function in part (a); the $x$-intercepts are $(-1, 0)$ and $(-6, 0)$. 