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Capital Investment and Non- Constant Marginal Cost of Capital

Mary Funck
Sam Houston State University

Steve Johnson
Sam Houston State University

Robert Stretcher
Sam Houston State University

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Abstract:

Business practice and prior research in capital budgeting establish that a firm's marginal cost of capital (MCC) is not constant across the scope of its investments. Capital budgeting decision methodology in textbooks and in practice, however, rarely addresses the full implications of capital budgeting decisions made under a non-constant MCC. Our research addresses this shortcoming by establishing a net present value (NPV) maximizing methodology that integrates a non-constant MCC. A Monte Carlo simulation is used to compare capital budgeting decisions made using the maximizing method to the decisions produced when utilizing a number of simplifying techniques. In addition, this research both quantifies the potential magnitude of investment errors and demonstrates the extent of shareholder wealth destruction that occurs when traditional assumptions are employed in place of using this optimizing approach.

I. Introduction

The most basic capital budgeting model, the Internal Rate of Return (IRR) method, is a simplified comparison of marginal benefits (returns on projects) to the marginal cost of funding. Under this model, a firm invests until the marginal cost equals the marginal benefit. Table 1 present's an example of using this basic evaluation model to assess three potential investment projects.

In order for a firm to reach a concrete conclusion regarding the accept/reject investment decision, assumptions are made concerning inputs to this model. One of the most common assumptions is that the marginal cost of capital (MCC) is constant across the scope of the investment opportunities under consideration. Implicit in this scenario is the understanding that a constant marginal cost of capital will be used to calculate the net present value (NPV) for each project and to arrive at a subsequent accept/reject investment decision.

(Insert Table 1 about here)

Non-constant MCC schedules, however, occur naturally in the business world. The MCC exhibits stepwise increases in WACC as relatively favorable financing sources are exhausted one-by-one. The reasons for this are numerous. Not all funds providers face the same risk, even if the business risk is the same for all projects in the firm’s capital budget. Creditors require varying rates of return as their ordinal claim against the firm's income differs or as their commitment of funds is secured or unsecured. Some lenders may be able to offer more favorable rates to a loan customer that provides the lender an opportunity to diversify their loan portfolio versus a lender already heavily invested in the firm's industry. Lenders often set a limit on the credit they are able to offer a borrower at a specified rate, simply because of the size of the borrower's loan compared to lender's loan portfolio. Internally generated equity financing is cheaper than external equity funding since external funding is accompanied by expenses due the investment bank for creating and issuing the stock. Even a best efforts issue, while reducing flotation costs, transfers the stock price risk to the firm. Any of the firm's funds providers may perceive greater risk as the capital budget becomes an increasingly larger proportion of the firm's current capitalization (i.e. a 5% expansion is not perceived to be as risky as a 100% expansion). Additionally, the capital structure used for funding the current capital budget may involve higher leverage than the current capital structure for the firm; a larger capital budget, then, would involve ever-increasing financial leverage for the firm as a whole.

To create a model, therefore, that more closely reflects the reality of the business environment, discarding the constant MCC assumption is necessary. Instead, the firm's actual cost of funding should be used – costs that increase as greater amounts of funding are obtained. The resulting complexity causes conceptual and practical challenges for educators, students, and practitioners. A simple example in which a firm has three projects under consideration highlights these challenges. These hypothetical projects' cash flows and resulting internal rates of return (IRR's) are shown in table 1. To maintain focus specifically on the complexity caused by the non-constant MCC, consider these three projects to be independent and non-divisible, with normal cash flows streams and the same life expectancy.

According to the IRR model criteria, a firm's investment opportunity schedule (IOS) dictates that potential projects be evaluated in order according to descending IRR (projects Victor, then Tango, and then Echo). In the proposed scenario, Projects Victor and Tango are both financed through capital raised at a marginal cost of 9.35% (since the combined capital investment for these projects falls below the $1,500,000 that can be raised at a marginal cost of 9.35%). The capital requirements associated with Project Echo cross several categories of funding and this project's marginal cost of capital (10.54%) becomes the weighted average of the costs associated with each of these funding categories. Refer to Figure 1 for a description of this calculation.

(Insert Figure 1 about here)

When the IOS schedule is superimposed (Figure 1) over the non-constant MCC, the decision implied by the IRR criterion is obvious: select Projects Victor and Tango, and reject Project Echo (accept projects where the IOS is greater than the MCC). The implied total NPV, when using the weighted MCC (wMCC) and accepting projects Victor and Tango, is $64,315.69 with a corresponding profitability index (PI) of 1.076.

A naive analyst might consider this to be the optimal choice since all three methods seem to agree. The only way to determine which investment decision is optimal, however, is to consider every possible rank-ordering for the three projects, and to assess the NPV for each permutation (Table 2). After examining all possible permutations and the resulting NPVs, it becomes evident that the IRR investment decision would be suboptimal. As shown in Figure 2, when the sequence of project evaluation is reordered (Projects Echo, Victor, and then Tango), the marginal cost of capital assigned to each project is adjusted, subsequently altering the accept/reject investment decision. Accepting project Echo and rejecting projects Victor and Tango results in a higher NPV than in the original ordering ($90,043.30).

(Insert Figure 2 about here)

In fact, even the profitability index measure can mislead (and disagree with the maximum NPV result) when the MCC is non-constant; the highest profitability index for accepted projects actually occurs in the first scenario when accepting projects Tango and Victor and rejecting project Echo.

(Insert Table 2 about here)

This simple example demonstrates that the presence of a non-constant MCC necessitates a more complete capital budgeting method than is provided by any one of the traditional methods. It also suggests a reason for why simplifying assumptions have been utilized instead of a more complex method: it is easier for professors to teach, it is easier for students to learn, and it is easier to apply in practice. Unfortunately, this approach potentially results in over- or under-investment and associated shareholder wealth destruction.

II. Literature Review

Some financial management textbooks include an exposition of non-constant MCC and the reasons it may exist, and they present the IRR criterion in relation to the MCC schedule (Block, et. al., 2011, pp. 344-48, and Brigham, et. al. 2008, Web Extension 11B, p.6). Some even hint that the rank-ordering of the projects is important, since the NPV is dependent on an appropriate discount rate for each project (Brigham, et. al. 2010, Hirschey 2009, p. 674). Few suggest addressing the evaluation of non-divisible marginal projects when the MCC breakpoint occurs at some level within that project, and none present a general application shareholder wealth maximizing method such as the one presented in this research.

Previous research exists concerning the difficulties of implementing the NPV rule without taking important details into consideration. Berkovitch and Israel (2004) discuss why the NPV criterion may not maximize NPV. Their argument is that "classical" information and agency considerations prevent the firm from implementing the optimal capital budgeting outcome. This differs from our model because Berkovitch and Israel formally model information and agency considerations, while our model includes only an increasing MCC curve while remaining agnostic regarding which forces (agency problems, information asymmetries, contracting difficulties, etc.) are responsible for driving the non-constant MCC.

Hirschleifer (1958) notes that when the firm has a non-constant MCC, the traditional method of ordering projects by their IRRs, then applying the NPV rule, may imply suboptimal project selection. He finds this is also true for when projects are not independent. His illustrations of these difficulties through the use of graphical utility function arguments are similar in spirit if not in detail to our investigation.

Stein (1997) investigates the notions of winner-picking and investment interdependence in a formal model. His winner-picking and investment interdependence correspond somewhat to the rank-ordering and resulting project interdependence in our model. However, he explicitly models the agency problem between a self-serving project manager and a self-serving HQ. In our model, we remain agnostic about the forces that drive the firm's non-constant MCC.

Higher cost of external funding is explored in a variety of studies - reviews of this literature can be found in Hubbard (1998) and Stein (2003). Campbell, Dhaliwal, and Schwartz (2012); Almeida and Campello (2007); and Rauh (2006), among others, address whether or not financing frictions influence real investment decisions. Other researchers contend that market frictions may cause the cost of capital raised externally to exceed the cost of internally generated cash flows (Guy and Stevens, 1994; Campbell, Dhaliwal, and Schwartz, 2012) due to information asymmetries, (Jensen and Meckling, 1976; Myers and Majluf, 1984) agency costs, (Jensen 1986), incomplete contracting, (Dybvig and Zender, 1991; Hirshleifer, 1993; Jaggia and Thakor, 1994), and taxes (Myers 1977).[1]

The non-constant MCC examined in this paper transcends the typical definitions for independent projects; we propose that when the rank order matters, even independent projects become interdependent. This is not a new idea; other aspects of specific business situations have similar effects. Williamson (1975) and Donaldson (1984) observe that in the internal financial market of large (usually diversified) firms, funds generated by one project are not immediately reinvested in the same project. Multiple projects compete for the funds generated by a single project. As a result, projects that have no relationship with each other except for existing inside the same firm become interdependent. One example of this phenomenon is illustrated by Lamont (1997), who documents how oil companies cut investment across the board in response to the oil price decline of 1986, including investment in non-oil-related projects. A second example is reported by Shin and Stulz (1996). They examine multi-division firms and find that investment in relatively small divisions is strongly related to the cash flows of other, larger divisions.

The interdependency of the investment opportunity schedule and the marginal cost of capital is also well recognized as it relates to budget constraints. Weingartner (1967) asserts that "…we have demonstrated that the common criteria for investment decisions are not appropriate tools for choosing among investments when there are limits to borrowing at a given rate of interest. In fact, we may assert that these criteria are not tools of capital *budgeting* (emphasis in the original) at all since they cannot be used to decide among investments within budget limitations of any kind" (pp. 177-178). In 1965, Baumol and Quandt noted that in the 1960's, computing power sufficient to solve capital budgeting problems that involved capital rationing was simply not available: "If budgets are fixed and the firm has under consideration a sizeable set of investment projects the number of combinations which the company can afford, and should therefore examine, is likely to be astronomical." In a footnote the authors noted that if 20 projects were available and the firm's capital constraint only allowed them to take 5 projects, the number of combinations would be 15,504 projects. Today, automated solutions are certainly possible, yet still complex to program.

III. Motivation

The presence of a non-constant MCC, in many cases, necessitates an assessment of the net present values of all possible rank-orderings to identify the maximum NPV solution. Similar to

Baumol and Quandt's (1965) statement above, large numbers of possible permutations result from just a few projects. For four projects, 24 permutations result. For five projects, 120 permutations result. For ten projects, 3,628,800 permutations must be assessed. Some firms consider even more projects in a single capital budgeting cycle. Modern computing capability allows us to automate the process of creating and evaluating all possible permutations, although the programming itself is still cumbersome and large numbers of projects may tax computing facilities and personnel.

The prominent question among our colleagues in academia and business is whether or not taking on our complex solution process is worthwhile, the traditional perception being that using informed assumptions of constant MCC should not result in suboptimal decisions, at least not appreciably so. The questioning of the need for our more complex method becomes even more pronounced when there are no other factors present that are typically blamed for the breakdown of capital budgeting measures: budget constraints, differences in business risk, nonconventional cash flow patterns within projects, unequal lives, mutual exclusivity, or the absence of other phenomena. These discussions motivated this study.

For a specified non-constant MCC, and a specified group of otherwise independent, non-divisible projects,[2] would assuming a constant MCC (or perhaps using a weighted average of the MCC levels across the scope of investment) really mislead to any significant degree, and if so, to *what* degree? Or would simply using the IRR method, independent of the MCC, return faulty decisions? We investigate the questions by conducting a simulation, varying the projects relating to a specified non-constant MCC. The output of interest would include the proportion of trials where suboptimal decisions result (from using an assumed constant MCC, a weighted average MCC, or the IRR method), the magnitude of the destruction of wealth from the suboptimal decisions, and the magnitude of the over-investment or under-investment (CF_0) from the sub-optimal decisions. Also of interest is the extent of inclusion or exclusion errors; that is, when a project that should have been rejected was accepted (inclusion), or a project that should have been accepted was rejected (exclusion).

IV. Monte Carlo Simulation

In order to model the dynamics of the method, a stepwise cost of capital schedule was specified. The schedule reflects five points of discontinuity (breakpoints), sufficient for a retained earnings limit and limits on four debt sources (Figure 3). The steps emulate a relatively heavy use of debt, but provide an acceptably diverse set of WACC levels, from 6.28% to 19.5%. Each iteration of our simulation will superimpose all possible permutations of five projects onto this MCC breakdown, for ten thousand iterations.

(Insert Figure 3 about here)

A fixed total initial investment (CF_0) of $10 million was used in order to make the totals of the CF_0 on five projects equal to the total of all other groups of five projects (in each of the 10,000 iterations). Project CF_0's were determined by generating five random integers between the bounds of 100 and 700 (creating a uniform distribution). Those five integers were summed, and each was converted into a weight (fraction). The CF_0 for each of the five projects was calculated by taking each of the five weights times $10 million. This produced a plausible variety of project CF_0's for each iteration, with the initial investment for the five projects totaling $10 million each time.

For each iteration, each of the five projects was assigned a randomly determined lifespan. A minimum life was specified at five years, and the maximum life was specified at forty years.

For each project, a set of cash inflows was generated. The algorithm used was designed to generate varied but typical normal cash inflow patterns for projects. A appropriate factor was provided by using Modified Accelerated Cost Recovery System (MACRS) depreciation rates per year for the appropriate project Asset Depreciation Range (ADR) midpoint.[3] The resulting cash inflow patterns approximated the gradual tapering of normal project cash inflows as the declining MACRS rates cycled. The resulting cash inflow patterns, we reasoned, were too uniform to provide an acceptable level of variation for projects of similar initial cash outlay and similar ADR midpoints. To diminish the uniformity, randomly generated bounded scaling factors based on project length were specified and used to create variability in the cash inflows, until the factor appeared to consistently result in a more acceptable variation in the cash inflow patterns for similar projects, but did not disrupt the pattern to the point of creating cash inflow pattern instability. The ultimate goal of the cash flow generation method was to achieve reasonably bounded IRR results for the projects in each iteration. Examples of resulting cash inflow patterns appear in Figure 4.

(Insert Figure 4 about here)

Each iteration of our simulation represents a five-project capital budgeting decision. We adopt the capital budgeting technique described in section 1 to derive the following for each iteration:

- 120 (5!) project permutations
- Weighted marginal cost of capital for each project in each permutation
- Each project's NPV using the weighted MCC *(wMCC)* as its cash inflow discount rate
- Using the NPV as a criterion:
  - A list of the resulting accepted projects
  - The total NPV of the accepted projects
  - The total CF_0 for the accepted projects
- Identify the permutation with the highest total NPV (the optimal project permutation),
- Record the optimal permutation's total NPV for accepted projects, and its total initial investment (CF_0)

The IRR method is perhaps the most likely method that may be used in a manager's attempt to avoid the complexity of the NPV maximizing methodology. Therefore, for comparison purposes, we identify the permutation that represents the IRR methodology (the permutation that ranks projects from highest to lowest IRR) and record the following for that permutation:

- Weighted marginal cost of capital for each project in each permutation
- Each project's NPV using the wMCC as its cash inflow discount rate
- Using the NPV as a criterion:
    - A list of the resulting accepted projects
    - The total NPV of the accepted projects
    - The total CF_0 for the accepted projects

We then compare the NPV and CF_0 for the optimal permutation to the NPV and CF_0 for the permutation representing the decision of the IRR method, recording the differences. We also record the inclusion errors (projects accepted in the IRR permutation but not in the optimal permutation) and the exclusion errors (projects not accepted in the IRR permutation but accepted in the optimal permutation).

Another likely attempt by a manager to avoid the complexity of our methodology is to simply assume some arbitrary discount rate to apply to all projects under consideration (bearing in mind that our study assumes all projects have equal business risk). In order to assess potential decision errors, we record the NPV, CF_0, and inclusion and exclusion errors for a discount rate representing the weighted average of the WACC levels across our $10 million possible investment scale (13.5%), as well as the same data assuming arbitrary rates close to the weighted average rate (we chose 11%, 13%, and 15%).

For each iteration, the optimal decision is thus compared to: 1) the IRR decision, 2) the decision resulting from assuming a constant discount rate based on the weighted average, and 3) the decisions resulting from simply assuming arbitrary constant discount rates. If these alternative methods result in optimal decisions, it is rational, of course, to avoid the complexity of the optimizing method involving every possible permutation. If there are errors, however, it is our intent to summarize the magnitude of those errors, their implications for possible wealth destruction, and the level of over- and/or under- investment, exclusive of the traditional explanations for the breakdown of capital budgeting methodology.

IV. Results

A large number of iterations of the Monte Carlo simulation (ten thousand) were run to reduce simulation error. The simulation was repeated multiple times, with a consistent result each time.

Over- and Under - Investment

For each iteration, the difference between the initial cash flow (CF_0) implied by the optimal solution was compared to the initial cash flow implied by each of the five comparison methods. Each difference was aggregated into a frequency distribution for each comparison method to highlight the investment error. The results are presented in Figure 5.

The IRR method (Panel A) resulted in an NPV maximizing investment level in 574 out of the 10,000 iterations (5.74%). Errors occurred in the remainder of the iterations (94.26%). The errors were distributed in a fairly normal distribution on either side of the zero-error column, smaller errors more common than larger errors, for both over-investment and under-investment. Under-investment (bins with negative CF_0 differences) was slightly more probable than over-investment (50.92% and 43.34% respectively).

As we might expect, use of arbitrary constant rates (shown in Panels B thru D) results in investment errors to a larger extent, with lower rates resulting in greater degrees of over-investment and vice versa. The weighted MCC rate (Panel E) yields substantial errors in both over- and under-investment (5.46% and 22.05% respectively), but yields no investment error in 249 iterations (2.49%). All of the arbitrary rate specifications and the weighted MCC rate resulted in noticeably skewed investment error distributions.

(Insert Figure 5 about here)

Net Present Value Error

For each iteration, the difference between the NPV implied by the optimal solution was compared to the NPV implied by each of the five comparison methods. Each difference was aggregated into a frequency distribution for each comparison method to highlight potential wealth destruction. The results are presented in Figure 6. The IRR method resulted in a maximizing decision with a probability of 1.05% (occurring 105 out of 10,000 iterations). As the frequency distribution indicates (Panel A), the frequency of larger errors dominated that of smaller errors. The greater insight is that the IRR and NPV-maximizing method disagree as to the optimal permutation in 98.95% of our iterations. Interestingly, the IRR method results in no inclusion or exclusion errors in 565 iterations, but only matches the optimal NPV in 105 iterations. This occurs because in 460 iterations (565-105), the NPV of the accepted projects is calculated in a suboptimal rank-ordering. In those 460 iterations, management would get a 'mulligan' of sorts- even though the methodology was misleading, the right projects would have been accepted but would have been valued incorrectly.

Using either arbitrary discount rates or a weighted average of the MCC schedule (Panels B thru E) results in what we might expect: lower discount rates will yield higher NPVs, and vice-versa. In the case of the 11% arbitrary rate, there are actually NPV's in a few cases that exceeded the NPV produced by the optimizing method. This does not mean that it is a superior decision - it means that the low, incorrect discount rate resulted in false positives. This is also reflected in the inclusion and exclusion errors in the next section; a lower rate tends to increase the likelihood of accepting a project that should have been rejected, and decrease the likelihood of rejecting a project that should have been accepted. The meaning of the NPV when the wrong discount rate is applied is thus in question. Similarly, assuming a higher constant rate shifts the distribution of NPV's lower, increasing the likelihood of rejecting a project that should have been accepted, and decreasing the likelihood of accepting a project that should have been rejected. Another result of note is that the NPVs produced using the three arbitrary constant discount rates never match the NPV of the optimizing method (although that occurrence is not impossible, it is extremely unlikely).

(Insert Figure 6 about here)

Inclusion and Exclusion Errors

For each iteration, the accepted and rejected projects implied by the optimal solution were compared to the accepted and rejected projects implied by each of the five comparison methods, producing a count of inclusion and exclusion errors. The number of inclusion/exclusion errors was concatenated and each pairing was aggregated into a frequency matrix for each comparison method to highlight the potential project selection errors. The results are presented in Table 3 and illustrated in Figure 7. For any given iteration, the IRR method resulted in no more than three inclusion errors and no more than two exclusion errors, and as few as no errors. The arbitrary rate comparisons form a pattern we might expect; the lower rates (11 and 13) have more inclusion errors (1-4 errors) often paired with no or few exclusion errors. The 15% arbitrary rate returned few inclusion errors, but two and three exclusion errors were common. The weighted average MCC rate returned zero to three inclusion and zero to three exclusion errors, one and two errors of either kind being more common. A complete breakdown of totals per category can be referenced in table 3.

(Insert Table 3 about here)

(Insert Figure 7 about here)

V. Discussion and Conclusion

This paper establishes a net present value maximizing methodology that accounts for the presence of a non-constant marginal cost of capital. A simulation was run in order to determine, for a specific marginal cost of capital schedule, whether conceptual and decision errors result from traditional simplifying assumptions and if so, to what magnitude. Our results indicate that over- and under- investment, wealth destruction, and inclusion and exclusion errors are common results of using a simplification technique rather than employing our maximum NPV criterion.

Our results have potential impact on a wide variety of research thrusts. We provide an (simpler) alternative explanation for over- and under-investment than those forwarded previously. The investment error occurs simply because of the presence of a non-constant MCC, without explicitly modeling information asymmetries, agency theory, contracting, multiple divisions, or explicit capital rationing. The result also transcends typical phenomena blamed for breakdowns in capital budgeting decision criteria: budget constraints, differences in business risk, non-normal cash flow patterns within projects, unequal lives, mutual exclusivity, etc. The breakpoints in a stepwise MCC emulate to a degree a budget constraint, but without an absolute barrier.

The magnitude of error is hard to ignore, and that the probability of error is substantial. We suspect that accepting these errors would be intolerable for any conscientious financial manager. Theorists, researchers, professors, students, financial managers and funds providers have a significant stake in whether or not this method is developed in the literature, included in textbooks, and practiced in the business world.

1. Sometimes firms ration capital; they choose to pass up positive NPV projects. There is evidence that this is at least partly due to the increased cost of external financing due to asymmetric information and agency problems (Thakor 1989, 1993).

2. Infinitely divisible projects, while theoretically possible, are rarely encountered in the business environment. Finitely divisible projects, such as a fleet of ten trucks, could just as easily be parsed into ten individual non-divisible projects, unless all ten trucks were needed to effectively serve a market, in which case the fleet would be a non-divisible project. Given this, we chose to establish our simulation using non-divisible projects only.

3. MACRS depreciation schedules may be accessed via IRS publication 946. The rates are presented in Appendix 1. The Asset Depreciation Range Classes and ADR midpoints appear in Appendix B of IRS Publication 946, but are not reproduced here.

References

Almeida, H., and M. Campello, 2007, "Financial Constraints, Asset Tangibility, and Corporate Investment," Review of Financial Studies 20:1, 429–60.

Baumol, William J. and Richard E. Quandt, 1965, " Investment and Discount Rates under Capital Rationing - A Programming Approach," The Economic Journal, Vol. 75, No. 298, June, 317-329.

Berkovitch, Elazar and Ronen Israel, 2004, "Why the NPV Criterion Does Not Maximize NPV," Review of Financial Studies, Vol. 17, No. 1, Spring, 239-255.

Block, Stanley B., Geoffrey A. Hirt, and Bartley R. Danielson, 2011, Foundations of Financial Management 14th edition, McGraw-Hill Irwin, New York, NY.

Brigham, Eugene, and Michael Ehrhardt, 2008, Financial Management: Theory and Practice 12th edition, Thompson Southwestern, Mason, OH.

Campbell John L., Dan S. Dhaliwal, and William C. Schwartz Jr., "Financing Constraints and the Cost of Capital: Evidence from the Funding of Corporate Pension Plans," Review of Financial Studies, Volume 25, Issue 3, 868-912.

Donaldson, Gordon, 1984, Managing Corporate Wealth, Praeger Publishers, New York.

Dybvig, P. H., and J.F. Zender, 1991, “Capital Structure and Dividend Irrelevance with Asymmetric Information,” Review of Financial Studies, Vol. 4, No. 1, 201-220.

Guy, V. and G. Stevens, 1994, "Internal Funds and the Investment Function," Southern Economic Journal, Vol. 60, No. 3, January, 551-563.

Hirshleifer, D., 1993, "Managerial Reputation and Corporate Investment Decisions," Financial Management, Summer, 145-160.

Hirshleifer, J., 1958, "On the Theory of Optimal Investment Decision." Journal of Political Economy, Vol. 66, No. 4 August, pp. 329-352.

Hubbard, G., 1998, "Capital-market Imperfections and Investment," Review of Economics and Statistics, 36, 193–225.

Jaggia, Priscilla B., and Anjan V. Thakor, 1994, "Firm-Specific Human Capital and Optimal Capital Structure," International Economic Review , Vol. 35, No. 2 May, 283-308.

Jensen, M., and W. Meckling, 1976, "Theory of the Firm: Managerial Behavior, Agency Costs, and Capital Structure," Journal of Financial Economics 3:11–25.

Jensen, Michael C., 1986, "Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers," The American Economic Review, Vol. 76, No. 2, Papers and Proceedings of the Ninety-Eighth Annual Meeting of the American Economic Association, May, 323-329.

Lamont, Owen, 1997, "Cash Flow and Investment: Evidence from Internal Capital Markets," Journal of Finance 52, 83-109.

Myers, Stewart C., 1977, " Determinants of corporate borrowing," Journal of Financial Economics, Volume 5, Issue 2, November, 147–175.

Myers, Stewart C., and Nicholas S. Majluf, 1984, "Corporate Financing and Investment Decisions When Firms Have Information that Investors Do Not Have," Journal of Financial Economics, June, 187-221.

Rauh, J., 2006, "Investment and Financing Constraints: Evidence from the Funding of Corporate Pension Plans," Journal of Finance 61: 33–71.

Shin, Hyun-Han, and Rene M. Stulz, 1996, "An Analysis of the Divisional Investment Policies of Diversified Firms," NBER Working paper No. 5639.

Stein, Jeremy C., 1997, "Internal Capital Markets and the Competition for Corporate Resources," The Journal of Finance, Vol. 52, No. 1 March, 111-133.

Stein, J., 2003, "Agency, Information, and Corporate Investment," In G. Constantinides, M. Harris, and R. Stulz (eds.), Handbook of the Economics of Finance. Amsterdam: Elsevier Science B.V.

Thakor, A.V., 1989, "Strategic Issues in Financial Contracting: An Overview," Financial Management Summer, 34-58.

Thakor, A.V., 1993, "Corporate Investments and Finance," Financial Management, Vol. 22, No. 2 Summer, 135-144.

Weingartner, H., 1967. Mathematical Programming and the Analysis of Capital Budgeting Problems. Markham Publishing Company, Chicago.

Williamson, Oliver E., 1975, Markets and Hierarchies: Analysis and Antitrust Implications Collier Macmillan Publishers, Inc., New York.

Table 1. Sample Project Cash Flows

Project Victor

ICO $510,000

IRR 0.107728

year	CF	PVIF	PV
1	$74,550.00	0.902748703	$67,299.92
2	$77,380.00	0.814955221	$63,061.24
3	$75,490.00	0.735699769	$55,537.98
4	$73,600.00	0.664152013	$48,881.59
5	$71,710.00	0.599562368	$42,994.62
6	$68,880.00	0.54125415	$37,281.59
7	$66,050.00	0.488616482	$32,273.12
8	$64,170.00	0.441097896	$28,305.25
9	$64,170.00	0.398200554	$25,552.53
10	$64,170.00	0.359475033	$23,067.51
11	$64,170.00	0.32451562	$20,824.17
12	$64,170.00	0.292956055	$18,798.99
13	$64,170.00	0.264465699	$16,970.76
14	$64,170.00	0.238746067	$15,320.34
15	$64,170.00	0.215527702	$13,830.41
			$510,000.00
			$0.00 NPV

Project Echo

ICO $1,420,000

IRR 0.1047255

year	CF	PVIF	PV
1	$204,260.00	0.905202243	$184,896.61
2	$212,010.00	0.819391101	$173,719.11
3	$206,840.00	0.741714663	$153,416.26
4	$201,670.00	0.671401776	$135,401.60
5	$196,500.00	0.607754394	$119,423.74
6	$188,740.00	0.550140641	$103,833.54
7	$180,990.00	0.497988542	$90,130.95
8	$175,820.00	0.450780345	$79,256.20
9	$175,820.00	0.40804738	$71,742.89
10	$175,820.00	0.369365404	$64,941.83
11	$175,820.00	0.334350392	$58,785.49
12	$175,820.00	0.302654725	$53,212.75
13	$175,820.00	0.273963736	$48,168.30
14	$175,820.00	0.247992588	$43,602.06
15	$175,820.00	0.224483447	$39,468.68
			$1,420,000.00
			$0.00 NPV

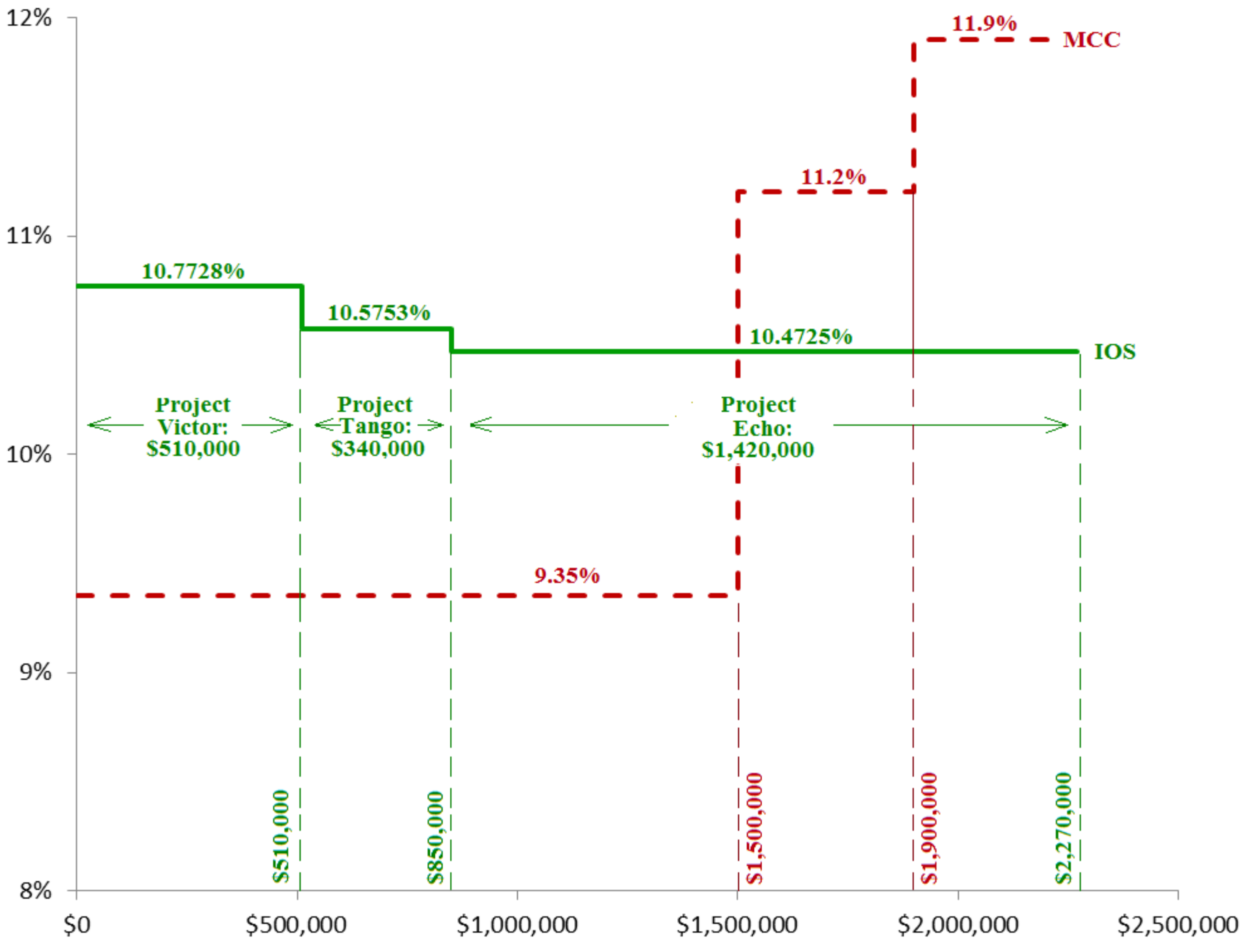
Project Tango

ICO $340,000

IRR 0.1057529

year	CF	PVIF	PV
1	$48,100.00	0.904361212	$43,499.77
2	$54,510.00	0.817869202	$44,582.05
3	$49,700.00	0.739649183	$36,760.56
4	$48,100.00	0.668910031	$32,174.57
5	$47,300.00	0.604936287	$28,613.49
6	$46,490.00	0.547080913	$25,433.79
7	$44,890.00	0.494758758	$22,209.72
8	$44,090.00	0.44744063	$19,727.66
9	$43,290.00	0.40464795	$17,517.21
10	$41,680.00	0.365947911	$15,252.71
11	$40,880.00	0.330949096	$13,529.20
12	$39,280.00	0.299297526	$11,756.41
13	$39,280.00	0.270673073	$10,632.04
14	$39,280.00	0.244786229	$9,615.20
15	$39,280.00	0.22137517	$8,695.62
			$340,000.00
			$0.00 NPV

Figure 1. IRR Method

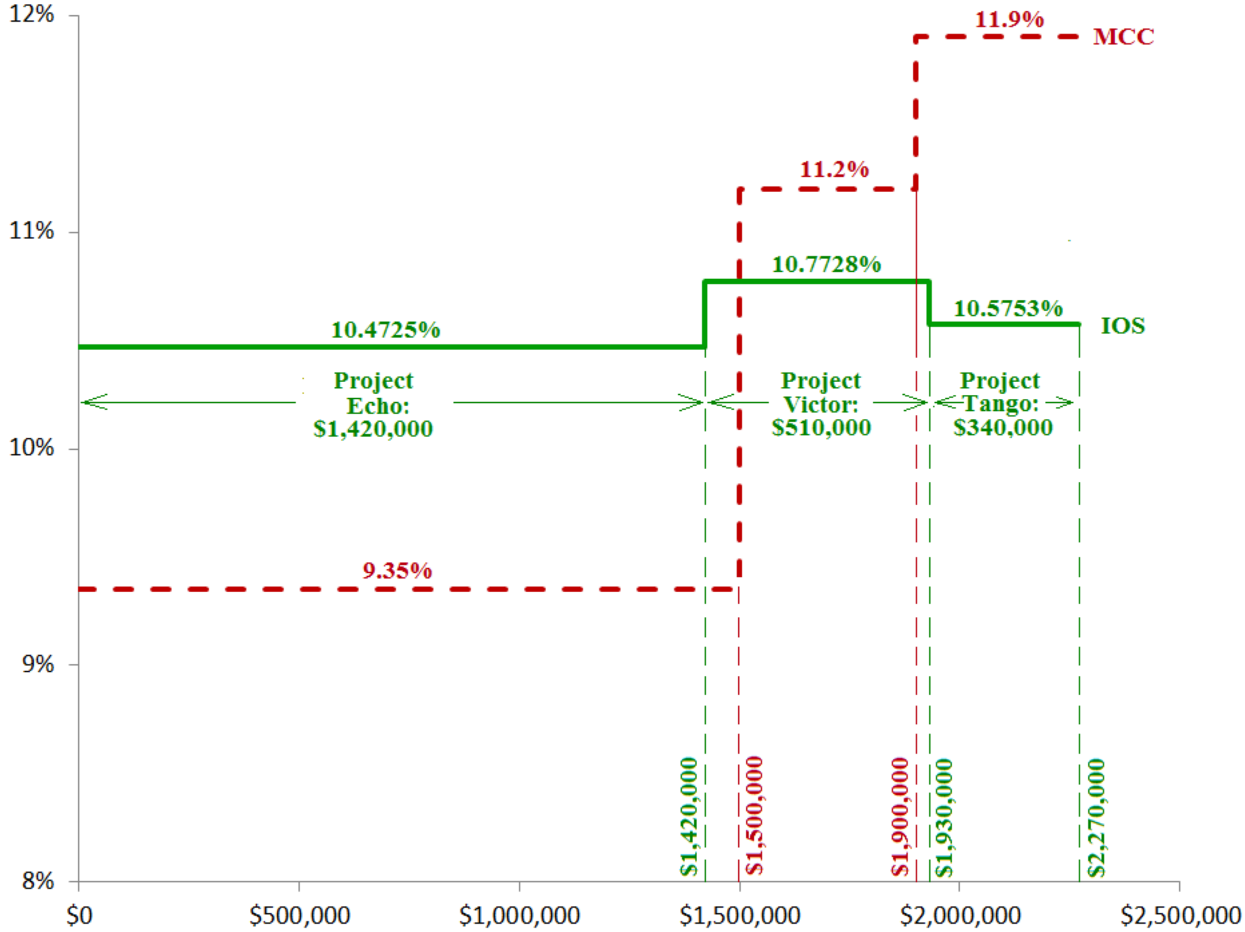


Project	***IRR***	***wMCC***	***Decision***	***NPV***	***PI***
Victor	*10.7728%*	*9.35%*	*accept*	*$41,114.00*	*1.080616*
Tango	*10.5753%*	*9.35%*	*accept*	*$23,201.69*	*1.068240*
Echo	*10.4725%*	*10.5356%*	*reject*	*-$4,796.97*	*.996622*
Total for Accepted Projects..				***$64,315.69***	***1.075666***

wMCC calculation for project Echo:

Investment	*MCC*	*Proportion*	*Weight*	*wMCC*
$650,000.00	*9.35%*	*650,000/1,420,000*	*.4577465*	*4.27993%*
$400,000.00	*11.20%*	*400,000/1,420,000*	*.2816901*	*3.15493%*
$370,000.00	*11.9%*	*370,000/1,420,000*	*.2605634*	*3.10070%*
		Project Echo Weighted MCC (wMCC).......		*10.53556%*

Figure 2. Alternative Rank-Ordering



Project	***IRR***	***wMCC***	***Decision***	***NPV***	***PI***
Echo	*10.4725%*	*9.35%*	*accept*	*$90,043.30*	*1.063411*
Victor	*10.7728%*	*10.95098%*	*reject*	*- $4,795.47*	*.990597*
Tango	*10.5753%*	*11.9%*	*reject*	*-$22,471.87*	*.933906*
Total for Accepted Projects..				***$90,043.30***	***1.063411***

Table 2. Summary for All Permutations

Project Rank Order	Accepted Projects	Total NPV	Total PI
Echo, Tango, Victor	Echo	$90,043.30*	1.063411
Echo, Victor, Tango	Echo	$90,043.30*	1.063411
Tango, Echo, Victor	Tango, Echo	$85,119.30	1.048363
Tango, Victor Echo	Victor, Tango	$64,315.69	1.075666*
Victor, Echo, Tango	Victor, Echo	$83,912.43	1.043478
Victor, Tango, Echo	Victor, Tango	$64,315.69	1.075666*

* *NPV Maximizing Solution*
** *PI Maximizing Solution*

Figure 3. Marginal Cost of Capital Schedule used in the Simulation

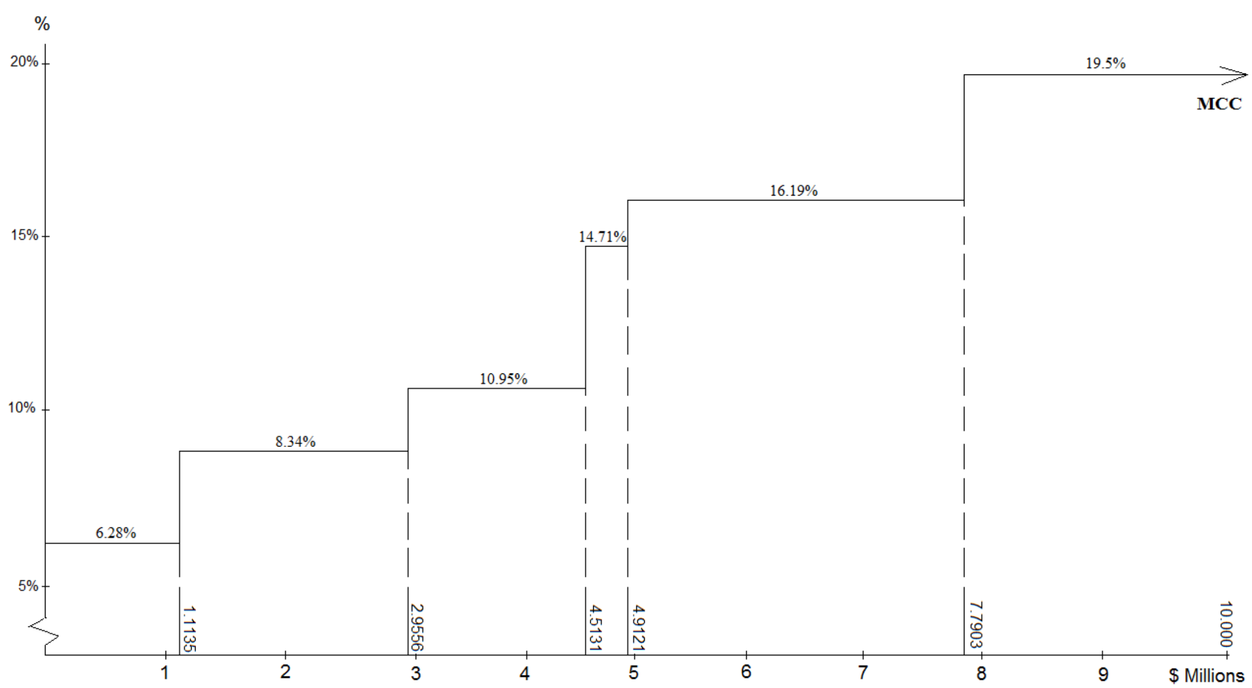


Figure 4. Examples of Project Cash Flow Patterns Generated

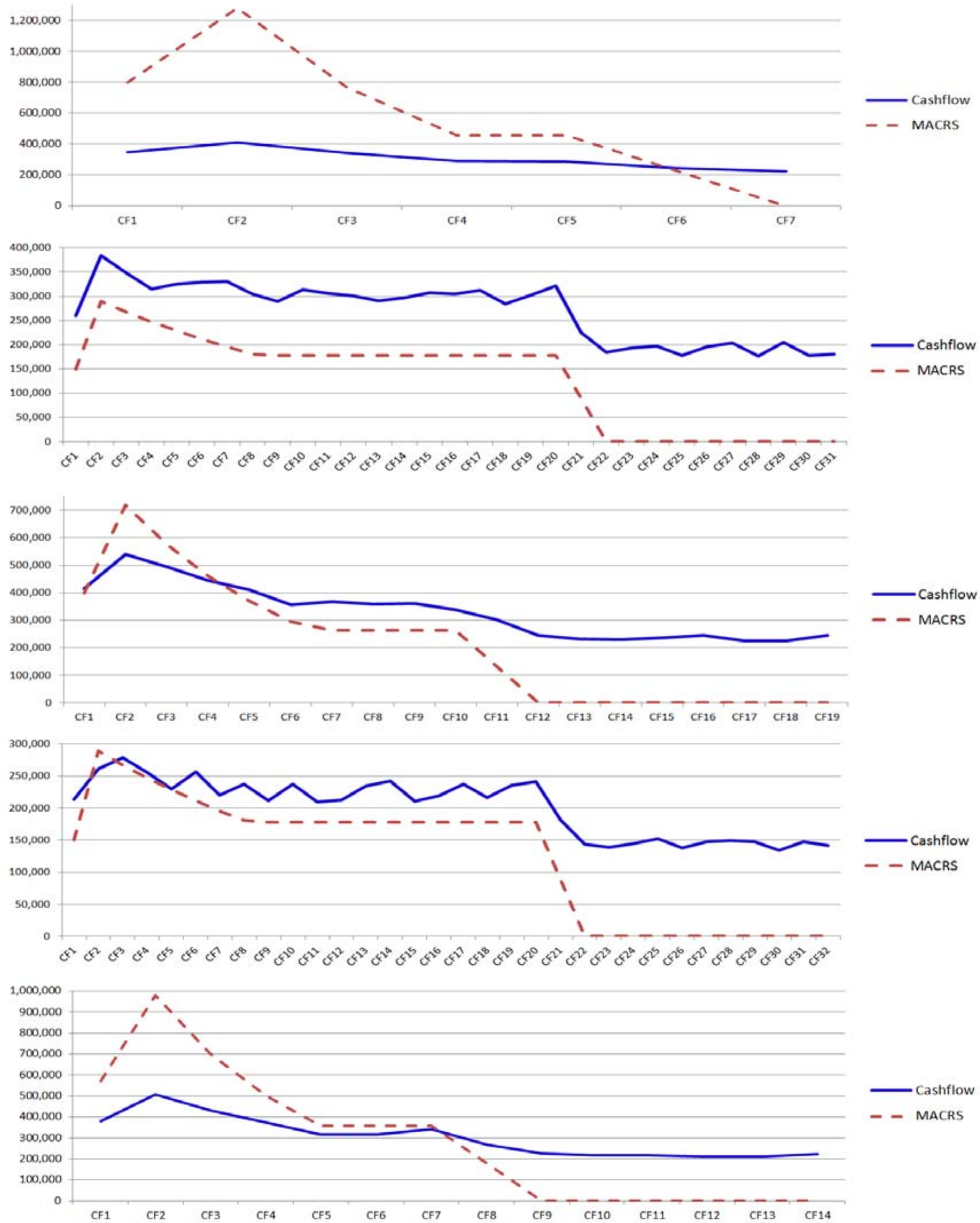


Figure 5. Difference in CF_0 per Each Comparison Method

(Bin identifiers represent CF_0 difference midpoints for each bin)

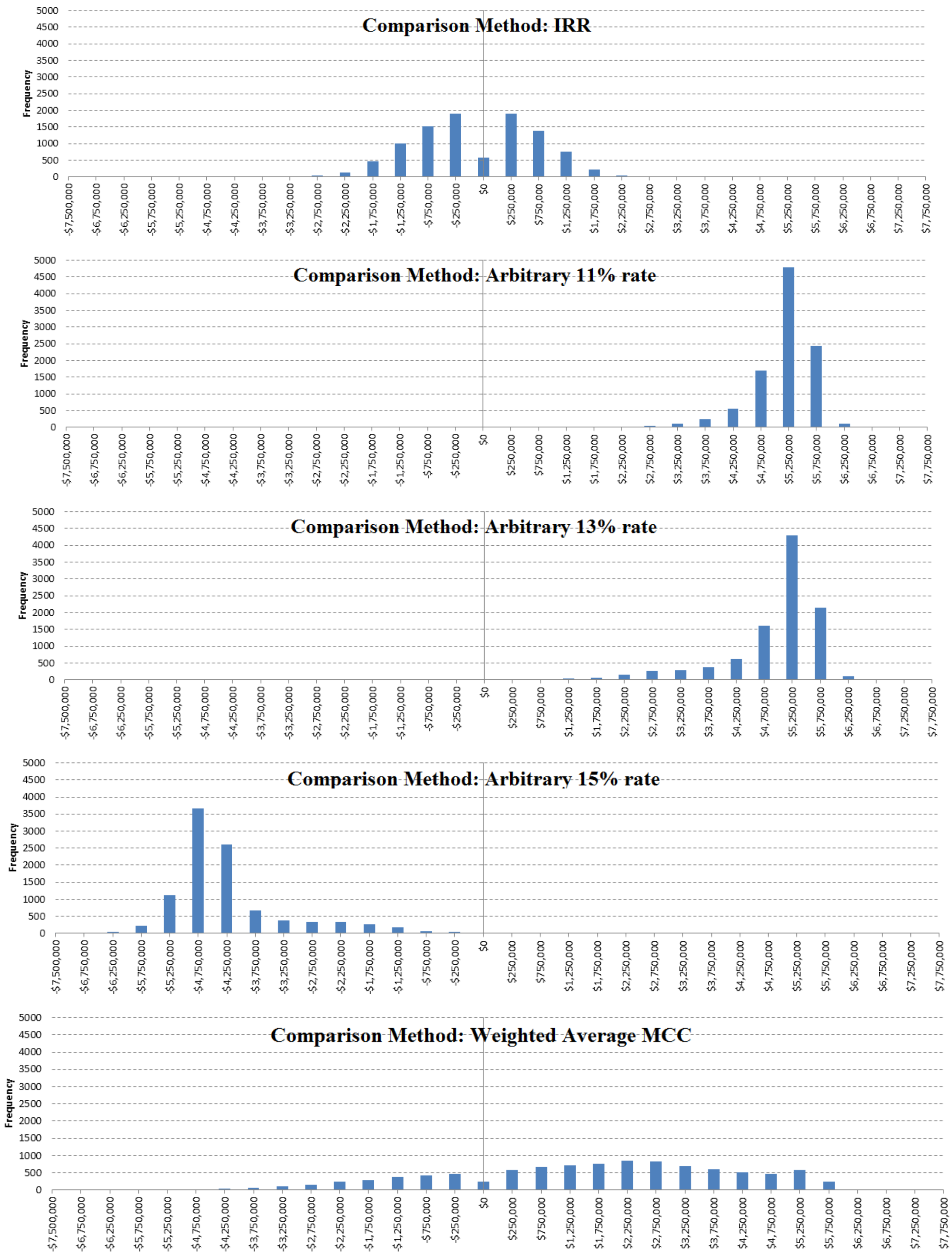


Figure 6. Difference in NPV per Each Comparison Method

(Bin identifiers represent NPV difference midpoints for each bin)

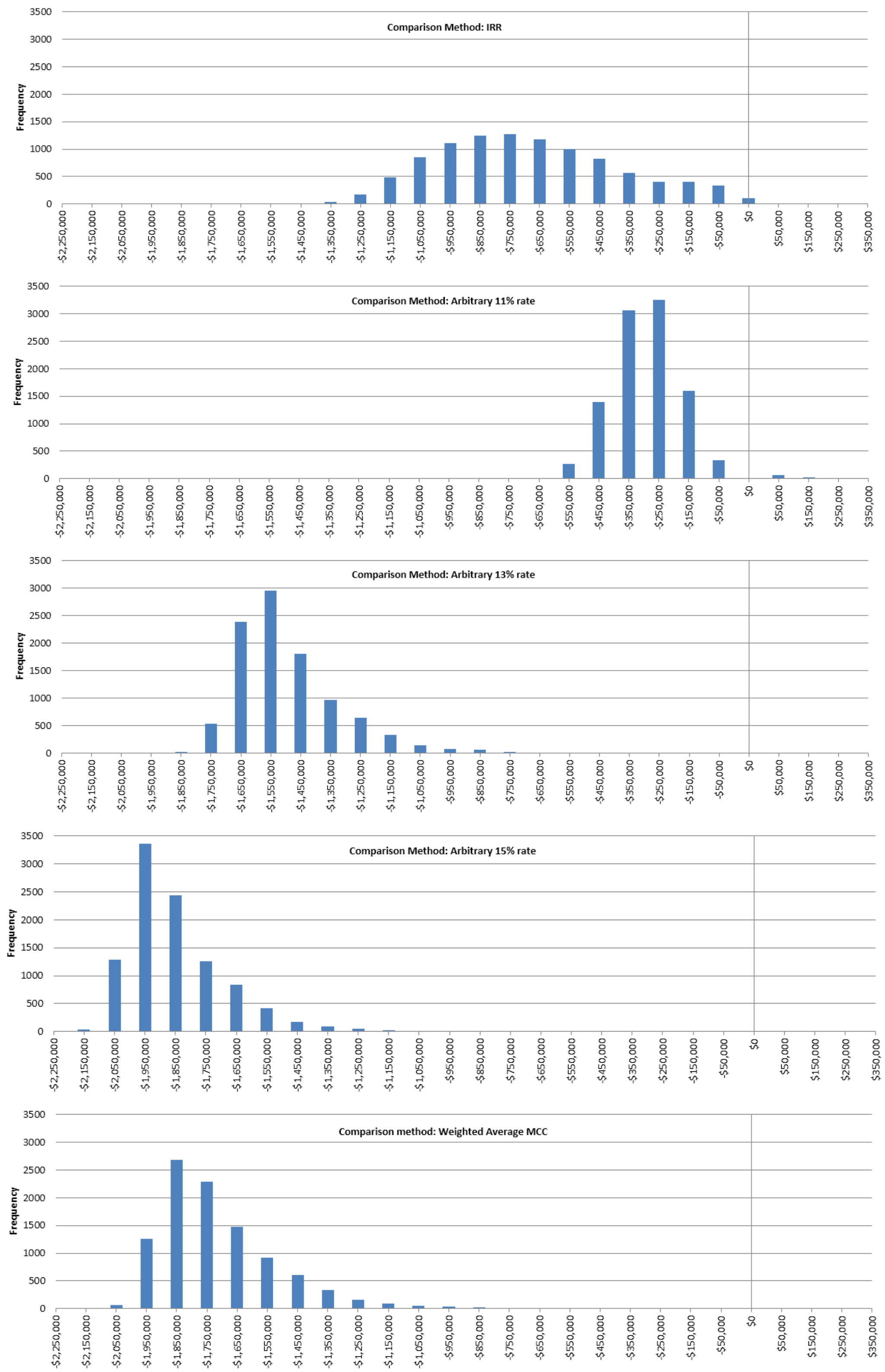


Table 3. Inclusion and Exclusion Errors

Comparison Method: IRR

Inclusion\Exclusion	0	1	2	3	4	5	
0	565	422	62	7	0	0	1,056
1	200	2535	2436	148	12	0	5,331
2	3	1112	1287	778	0	0	3,180
3	0	26	406	0	0	0	432
4	0	1	0	0	0	0	1
5	0	0	0	0	0	0	0
	768	4096	4191	933	12	0	10,000

Comparison Method: Arbitrary 11% Discount Rate

Inclusion\Exclusion	0	1	2	3	4	5	
0	0	0	0	0	0	0	-
1	196	0	0	0	0	0	196
2	5489	0	0	0	0	0	5,489
3	4266	0	0	0	0	0	4,266
4	49	0	0	0	0	0	49
5	0	0	0	0	0	0	0
	10000	0	0	0	0	0	10,000

Comparison Method: Arbitrary 13% Discount Rate

Inclusion\Exclusion	0	1	2	3	4	5	
0	12	1	0	0	0	0	13
1	555	24	1	0	0	0	580
2	5377	161	2	0	0	0	5,540
3	3781	44	0	0	0	0	3,825
4	42	0	0	0	0	0	42
5	0	0	0	0	0	0	0
	9767	230	3	0	0	0	10,000

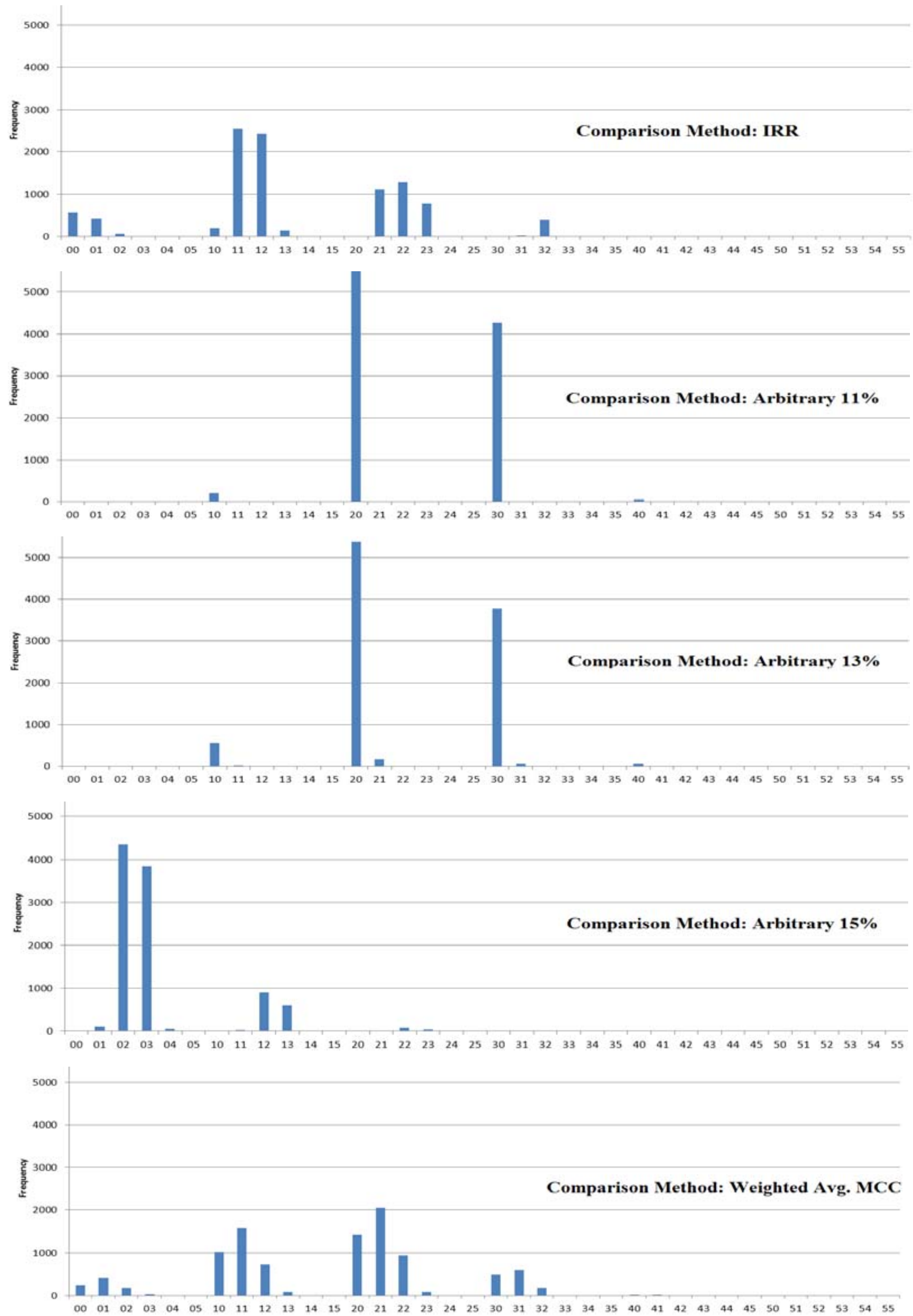
Comparison Method: Arbitrary 15% Discount Rate

Inclusion\Exclusion	0	1	2	3	4	5	
0	0	108	4339	3843	57	0	8,347
1	0	24	897	596	3	0	1,520
2	0	3	87	41	0	0	131
3	0	0	2	0	0	0	2
4	0	0	0	0	0	0	-
5	0	0	0	0	0	0	0
	0	135	5325	4480	60	0	10,000

Comparison Method: Weighted Average MCC

Inclusion\Exclusion	0	1	2	3	4	5	
0	248	404	178	25	0	0	855
1	1012	1575	727	82	0	0	3,396
2	1413	2060	927	82	0	0	4,482
3	492	590	175	0	0	0	1,257
4	3	7	0	0	0	0	10
5	0	0	0	0	0	0	0
	3168	4636	2007	189	0	0	10,000

Figure 7. Frequency of Inclusion/Exclusion Errors per Each Comparison Method



Appendix 1 – MACRS Depreciation Schedule

	DepreciationRate For Recovery Period					
Year	3-year	5-year	7-year	10-year	15-year	20-year
1	0.3333	0.2000	0.1429	0.1000	0.0500	0.03750
2	0.4445	0.3200	0.2449	0.1800	0.0950	0.07219
3	0.1481	0.1920	0.1749	0.1440	0.0855	0.06677
4	0.0741	0.1152	0.1249	0.1152	0.0770	0.06177
5		0.1152	0.0893	0.0922	0.0693	0.05713
6		0.0576	0.0892	0.0737	0.0623	0.05285
7			0.0893	0.0655	0.0590	0.04888
8			0.0446	0.0655	0.0590	0.04522
9				0.0656	0.0591	0.04462
10				0.0655	0.0590	0.04461
11				0.0328	0.0591	0.04462
12					0.0590	0.04461
13					0.0591	0.04462
14					0.0590	0.04461
15					0.0591	0.04462
16					0.0295	0.04461
17						0.04462
18						0.04461
19						0.04462
20						0.04461
21						0.02231