On metric dimension, strong metric dimension, and zero forcing number of graphs
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Abstract: Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$. The metric dimension $\dim(G)$ of a graph $G$ is the minimum number of vertices such that every vertex of $G$ is uniquely determined by its vector of distances to the chosen vertices. The strong metric dimension $\text{sdim}(G)$ of a graph $G$ is the minimum among cardinalities of all strong resolving sets: $W \subseteq V(G)$ is a strong resolving set of $G$ if for any $u, v \in V(G)$, there exists an $x \in W$ such that either $u$ lies on an $x - v$ geodesic or $v$ lies on an $x - u$ geodesic. The zero forcing number $Z(G)$ of a graph $G$ is the minimum cardinality of a set $S$ of black vertices (whereas vertices in $V(G) - S$ are colored white) such that $V(G)$ is turned black after finitely many applications of “the color-change rule”: a white vertex is converted black if it is the only white neighbor of a black vertex.

In this talk, we show that $\dim(T) \leq Z(T) \leq \text{sdim}(T)$ for a tree $T$ and that $\dim(G) - 1 \leq Z(G) \leq \text{sdim}(G)$ for a unicyclic graph $G$. Further, we show that $Z(G) \leq \text{sdim}(G) + 3r(G)$ for a connected graph $G$, where $r(G)$ is the cycle rank of $G$. We also discuss the effect of vertex or edge deletion on the metric dimension of graphs. We end with a proof of $\dim(T) - 2 \leq \dim(T + e) \leq \dim(T) + 1$ for any tree $T$ and an edge $e \in E(T)$.

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