1. For which of the following is \( y \) a function of \( x \)? Write “Yes” or “No” following the statement. A brief explanation or equation is required to defend your work.

(a) \( \{(2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (4.5, 2), (5.3, \sqrt{5})\} \)
(b) \( y^2 + x^2 = 4 \)
(c) \( xy = 0 \)

Solutions.

(a) \( \boxed{\text{YES}} \) \( \{(2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (4.5, 2), (5.3, \sqrt{5})\} \) IS a function since every one of the seven inputs has a unique output.
(b) \( \boxed{\text{NO}} \) \( y^2 + x^2 = 4 \) is NOT a function since (for example) the input \( x = 0 \) has two outputs, \( y = \pm 2 \). Indeed, the graph of \( y^2 + x^2 = 4 \) is a circle of radius 2 centered at the origin. It fails the vertical line test.
(c) \( \boxed{\text{NO}} \) \( xy = 0 \) is NOT a function since the input \( x = 0 \) has many possible outputs. Indeed, if \( x \) is zero then \( y \) can be anything.

2. Suppose \( f(x) = \sqrt{\frac{x-3}{7}} + 4 \) and \( g(x) = 7(x-4)^2 + 3 \).

Compute

(a) \( (f \circ g)(x) \)
(b) \( (g \circ f)(x) \)

Solutions.

(a) \( (f \circ g)(x) = x \)
(b) \( (g \circ f)(x) = x \)

3. Suppose \( f \) is the function \( \{(3, 1), (4, 2), (5, 2), (6, 7)\} \) and \( g \) is the function \( \{(1, 12), (3, 5), (4, 5), (7, 3)\} \)

(a) \( (f \circ g)(3) \)
(b) \( (f \circ g)(7) \)
(c) \( (g \circ f)(3) \)
(d) \( (g \circ f)(6) \)

Solutions.

(a) \( (f \circ g)(3) = f(g(3)) = f(5) = 2. \)
(b) \( (f \circ g)(7) = f(g(7)) = f(3) = 1. \)
(c) \( (g \circ f)(3) = g(f(3)) = g(1) = 12. \)
(d) \( (g \circ f)(6) = g(f(6)) = g(7) = 3. \)
4. Compute, algebraically, the inverse of each of the following functions.

(a) \( h(x) = 7(x - 4)^2 + 3 \)
(b) \( h(x) = x^2 - 12x + 36 \)

**Solutions.**

(a) To compute the inverse of \( h(x) = 7(x - 4)^2 + 3 \), set \( y = 7(x - 4)^2 + 3 \). Then swap inputs and outputs and solve for \( y \).

\[
x = 7(y - 4)^2 + 3.
\]

Subtract 3 from both sides and then divide by 7 to get

\[
\frac{x - 3}{7} = (y - 4)^2.
\]

Take the square root of both sides and then add 4 so that

\[
\sqrt{\frac{x - 3}{7}} + 4 = y.
\]

So

\[
h^{-1}(x) = \sqrt{\frac{x - 3}{7}} + 4.
\]

(b) To compute the inverse of \( h(x) = x^2 - 12x + 36 \), set \( y = x^2 - 12x + 36 \) and then recognize the righthand sides as \((x - 6)^2\). Swap inputs

\[
x = (y - 6)^2,
\]

take square roots and add 6 to both sides.

\[
\sqrt{x} + 6 = y.
\]

So

\[
h^{-1}(x) = \sqrt{x} + 6.
\]