Sample Mini-Exam 1C, MATH 1410
(Over lectures 1.1-1.6)
(SOLUTIONS)
HUMANS ONLY! Calculators are NOT allowed.

1. \( f(x) = x^2 + 5x + 8 \). Compute the “difference quotient” \( \frac{f(x+h) - f(x)}{h} \).
   Solution.
   \[
   \frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + 5(x+h) + 8 - (x^2 + 5x + 8)}{h} = \frac{x^2 + 2xh + h^2 + 5x + 5h + 8 - x^2 - 5x - 8}{h} = \frac{2xh + h^2 + 5h}{h} = \frac{h(2x + h + 5)}{h} = 2x + h + 5.
   \]

2. For each of the functions below, compute \( f(-x) \) and use your answer to determine if \( f(x) \) is an even function, an odd function or neither.
   (a) \( f(x) = \frac{100}{2x^3 + 6x} \).
   (b) \( f(x) = \frac{x}{x^4 - 16} \).
   Solutions.
   (a) If \( f(x) = \frac{100}{2x^3 + 6x} \) then \( f(-x) = \frac{-100}{-2x^3 - 6x} = -f(x) \) so this function is \text{ODD}.
   (b) \( f(x) = \frac{x}{x^4 - 16} \) then \( f(-x) = \frac{-x}{x^4 - 16} = -f(x) \) so this function is \text{ODD}.

3. Suppose \( f(x) = x^2 - 25 \) and \( g(x) = x - 5 \). Compute
   (a) \((f \circ g)(x)\)
   (b) \((g \circ f)(x)\)
   Solutions.
   (a) \((f \circ g)(x) = f(g(x)) = f(x - 5) = (x - 5)^2 - 25 = x^2 - 10x + 25 - 25 = x^2 - 10x\)
   (b) \((g \circ f)(x) = g(f(x)) = g(x^2 - 25) = (x^2 - 25) - 5 = x^2 - 30\)

4. Suppose \( f \) is the function \{ (3, 1), (4, 2), (5, 2), (6, 7) \} and \( g \) is the function \{ (2, 12), (3, 16), (4, 5), (7, 3) \}
   (a) \((f \circ g)(6)\)
   (b) \((g \circ f)(4)\)
   (c) \((f \circ g)(4)\)
   (d) \((g \circ f)(7)\)
   Solutions.
   (a) \((f \circ g)(6)\) does not exist (DNE) since 6 is not in the domain of \( g \).
   (b) \((g \circ f)(4) = g(f(4)) = g(2) = 12\)
(c) \((f \circ g)(4) = f(g(4)) = f(5) = 2\)
(d) \((g \circ f)(7)\) does not exist (DNE) since 7 is not in the domain of \(f\).

5. Compute, algebraically, the inverse of each of the following functions.

(a) \(h(x) = \sqrt{x^2 - 25}\)
(b) \(h(x) = 7(x - 4)^2 + 3\)
(c) \(h(x) = \frac{2x + 3}{7}\)
(d) \(h(x) = \frac{1}{3x + 2}\)
(e) \(h(x) = \frac{7x}{x - 6}\)

**Solutions.**

(a) To compute the inverse of \(h(x) = \sqrt{x^2 - 25}\), set \(y = \sqrt{x^2 - 25}\). Then swap inputs and outputs and solve for \(y\).

\[
x = \sqrt{y^2 - 25},
\]
square both sides
\[
x^2 = y^2 - 25,
\]
and then add 25 to both sides
\[
x^2 + 25 = y^2,
\]
and then take the square root of both sides
\[
\sqrt{x^2 + 25} = y,
\]
so that
\[
h^{-1}(x) = \sqrt{x^2 + 25}.
\]

(b) To compute the inverse of \(h(x) = 7(x - 4)^2 + 3\), set \(y = 7(x - 4)^2 + 3\). Then swap inputs and outputs and solve for \(y\).

\[
x = 7(y - 4)^2 + 3;
\]
add 3 to both sides
\[
x - 3 = 7(y - 4)^2,
\]
and then divide both sides by 7,
\[
\frac{x - 3}{7} = (y - 4)^2,
\]
and then take the square root of both sides
\[
\sqrt{\frac{x - 3}{7}} = y - 4,
\]
and finally add 4 to both sides
\[
\sqrt{\frac{x - 3}{7}} + 4 = y,
\]
so that
\[
h^{-1}(x) = \sqrt{\frac{x - 3}{7}} + 4.
\]
(c) To compute the inverse of \( h(x) = \frac{2x + 3}{7} \), set \( y = \frac{2x + 3}{7} \). Then swap inputs and outputs and solve for \( y \).

\[
x = \frac{2y + 3}{7};
\]

multiply both sides by 7,

\[
7x = 2y + 3,
\]

and subtract 3 from both sides,

\[
7x - 3 = 2y,
\]

and finally divide both sides by 2 so that

\[
\frac{7x - 3}{2} = y,
\]

and our final answer is

\[
h^{-1}(x) = \frac{7x - 3}{2}.
\]

(d) To compute the inverse of \( h(x) = \frac{1}{3x + 2} \), set \( y = \frac{1}{3x + 2} \) and swap variables

\[
x = \frac{1}{3y + 2};
\]

multiply both sides by the denominator

\[
x(3y + 2) = 1,
\]

multiply out the left side,

\[
3xy + 2x = 1;
\]

subtract 2x from both sides

\[
3xy = 1 - 2x
\]

and then divide both sides by 3x so that

\[
y = \frac{1 - 2x}{3x}.
\]

So

\[
h^{-1}(x) = \frac{1 - 2x}{3x}.
\]

(e) \( h(x) = \frac{7x}{x - 6} \) To compute the inverse of \( h(x) = \frac{7x}{x - 6} \), set \( y = \frac{7x}{x - 6} \) and swap variables

\[
x = \frac{7y}{y - 6}.
\]

multiply both sides by the denominator

\[
x(y - 6) = 7y
\]

multiply out the left side,

\[
xy - 6x = 7y
\]

get terms with \( y \) on the lefthand side

\[
xy - 7y = 6x,
\]
factor out $y$,

$$y(x - 7) = 6x,$$

and divide both sides by $x - 7$, solving, finally, for $y$.

$$y = \frac{6x}{x - 7}.$$

So

$$h^{-1}(x) = \frac{6x}{x - 7}.$$