Worksheet 1 on primitive counting
Numbers in primitive societies, computations with positional notation
MTH 4367, Spring 2012

(None of this material requires mathematics beyond the high school curriculum.)

1. Read through the material on primitive counting and primitive mathematical cultures in your textbook. Summarize that material in several paragraphs (200-500 words.)

Note: Anytime in this class that you are asked to “summarize” your textbook, you must include, in that summary, the author, title and edition of that textbook. And, of course, the summary must be in your own words. There is no reason to quote (or copy) any of the textbook material.

2. You are an anthropologist, studying an ancient civilization. Translate the following into numbers:
   (a) from the Jibaro Indians of Amazon rain forest the phrase mai wehe amukei literally means: I have finished both hands. What number does this phrase represent?
   (b) Greenland Inuit: other hand two.
   (c) Greenland Inuit: first foot one.
   (d) Bellacoola (British Columbia) one man less two.
   (e) Malinke of West Sudan, dibi means mattress.

3. Briefly explain the difference between a multiplicative numeral system and a positional numeral system.

4. In which numeral systems (simple counting, simple grouping, multiplicative, ciphered, positional) would one have been able to represent numbers between 0 and 1? Why?

5. What impact did the writing materials (papyrus, clay, bamboo, paper) have on the development of mathematics and our knowledge of it? How did the different writing materials influence the mathematical notation?

6. (From Burton, page 19, problem 5) Write the Ionian Greek numerals corresponding to
   (a) 385
   (b) 1472
   (c) 8888
   (d) 24789
   (e) 123457
   (f) 1234567

7. Use the English “ciphered” system (A = 1, B = 2, ... I = 9, J = 10, K = 20, ... mimicking the Greek ciphered system) to write out the number of
   (a) your first name
   (b) your middle name
   (c) your last name
   (d) your entire name (first, middle and last)
8. Write the modern equivalents of the following Mayan numerals. Assume the most significant digit is at the top.

(a) 

(b) 

9. Redo the previous problem by applying the “calendric adjustment” – using 360 – described in the class notes (or in Burton, page 9.)

10. Write the following in base 12.

(a) 40

(b) 2883

(c) 3259

(d) \( \frac{2}{3} \)

(e) \( \frac{1}{27} \)
Worksheet 2 on primitive counting
Egyptian mathematics and unit fractions
MTH 4367, Spring 2012

(None of this material requires mathematics beyond the high school curriculum.)

1. The choice of base used in positional notation is a human construct; it is not
intrinsic to mathematics. Write an essay (200-500 words) explaining why humans have used
certain bases. Your answers should include tentative explanations for the occurrence of bases five, ten, twelve, twenty
and sixty in the computations of early cultures.

2. Computations in base two and base sixteen occur today in our modern culture. Where? And why?
(A good answer should include a paragraph answer to the “why” part of this question.)

3. In the far future, we find an alien race which uses base 21 in their computations. Provide a variety
of possible explanations for their use of base 21.

4. (From Eves, p. 27, problem 1.1) Suggest possible explanations for the following linguistic phrase
for numbers.

(a) A translation of a Bible passage for a Papuan tribe in New Guinea had to use the following
phrase for number 38: “one man, both hands, five and three.”
(b) In British New Guinea the number 99 was expressed as “four men die, two hands come to an
end, one foot ends, and four.”
(c) The South American Kamayura tribe uses the phrase “peak-finger days” to mean “three days.”
(d) The Zulus of South Africa use phrases “taking the thumb” and “he pointed” for the numbers
six and seven, respectively.

5. (From Eves, p. 30, problem 1.8)

(a) Write 30125 in base 8.
(b) Can the following numbers be even for some integer base b? (Why? Why not?)
   i. 27b
   ii. 37b
   iii. 73b
   iv. 83b
(c) Find an integer base b such that
   i. 79 = 142b
   ii. 72 = 2200b
(d) What is the smallest integer base for which 301b represents a square?
(e) For which integers b is the number 121b a perfect square?

6. Write the fraction 2/7 in “radix” form (0.b₁b₂...bₖ...) in
(a) base five
(b) base ten
(c) base twelve
7. Write the fraction $\frac{1}{3}$ in “radix” form ($0.b_1b_2...b_k...$) in
   (a) base five
   (b) base ten
   (c) base twelve
   (d) base twenty
   (e) base sixty

8. Is it possible to have bases that are not integers? For example, could we have a base $b = \frac{3}{2}$? Or a base $b = \frac{7}{4}$?
   (a) What would $110_b$ represent in base $b = \frac{3}{2}$?
   (b) What would $1.\overline{1}_b$ represent in base $b = \frac{3}{2}$?
   (c) How might one write the integer 2 in base $b = \frac{3}{2}$?
Worksheet 3 on primitive counting
Babylonian mathematics
MTH 4367, Spring 2012

(Problems 1-9 do not require mathematics beyond the high school curriculum; problem 10 requires Taylor series from a second semester calculus course.)

1. Read through the material on the Babylonian and Egyptian cultures in your textbook. Summarize your understanding of these societies in several paragraphs (200-500 words.) Feel free to add additional thoughts from information found on the internet.

(Note: Continue to follow the expectations regarding summaries, as described in Worksheet 1.)

2. Describe the fractions which terminated in the Babylonian sexagesimal system.

3. Write out the modern equivalents of the following sexagesimal numbers.
   (a) 25, 53, 7; 24, 57
   (b) 8, 29, 44, 0, 47
   (c) 3; 8, 30
   (d) 3; 8, 29, 44, 0, 47
   (e) 3; 8, 29, 44, 0, 47, 25, 53, 7, 24, 57. What number does this approximate?

4. (from Dr. Dustin Jones's youtube video.) Write each of the numbers below in the sexagesimal system.
   (a) 52
   (b) 106
   (c) 240
   (d) 1000
   (e) 4000
   (f) 3636

5. (from Dr. Dustin Jones's youtube video.) Write each of the numbers below in the Babylonian sexagesimal system, using the Babylonian symbols.
   (a) 52
   (b) 106
   (c) 240
   (d) 1000
   (e) 4000
   (f) 3636

6. Write out the modern equivalents of the following Babylonian numbers:
   (a) 🦁
7. Write the following in sexagesimal notation.
   (a) 3259
   (b) 123.4
   (c) 133.24
   (d) one-seventh
   (e) one-eleventh
   (f) one-thirteenth

8. Using sexagesimal notation square the number 1; 24, 51, 10. (Do this work exclusively in sexagesimal notation! Do not give in to your base ten bias!)

9. Use linear interpolation to approximate the doubling time for interest compounded annually at ten percent. Put your final answer in sexagesimal notation.

10. (From Eves, p. 60, problem 2.7) The Babylonian approximation for the square root function was
    \((a^2 + h)^{1/2} \approx a + \frac{h}{2a}\).
    (a) Verify that \(a = \frac{4}{3}, h = \frac{2}{5}\) would give an approximation for \(\sqrt{2}\). What value does this give for \(\sqrt{2}\)?
    (b) Verify that \(a = 3, h = -1\) would give an approximation for \(\sqrt{8}\) and so, dividing this number by 2, we would have an approximation for the square root of 2. What approximation does this computation give for \(\sqrt{2}\)?
    (c) Use \(a = 2, h = 1\) to approximate \(\sqrt{5}\).

11. The Taylor series for \(f(x) = \sqrt{x}\) expanded about the value \(x = a^2\) is
    \[f(x) \approx f(a^2) + f'(a^2)(x - a^2) = a + \frac{1}{2a}(x - a^2).\]
    If we write \(x = a^2 + h\) then we achieve the formula
    \[\sqrt{a^2 + h} \approx a + \frac{h}{2a}\]
    used by the Babylonians. But we can do better:
(a) Write out the next term of the Taylor series for \( f(x) = \sqrt{x} \) expanded about \( a^2 \) and so get a better formula for the square root of \( a^2 + h \). (This formula will include a term involving the second derivative of \( f(x) \) and \( h^2 = (x - a^2)^2 \).)

(b) Use this better formula found in part (a) to re-do the previous problem (problem 9) and so get better approximations for \( \sqrt{2}, \sqrt{3}, \) and \( \sqrt{5} \).
1. Summarize what we have learned from the following papyri:
   (a) The Moscow papyrus
   (b) The Rhind papyrus

2. (Burton, page 18, problem 1.) Write each of the numbers below in Egyptian hieroglyphics.
   (a) 1492
   (b) 1999
   (c) 12321
   (d) 70807
   (e) 123456
   (f) 3040279

3. (Burton, page 18, problem 2) Write each of these Egyptian numbers in our modern system.
   (a) \[ \begin{array}{c}
   ||||| \\
   \square\square\square\square
   \end{array} \]
   (b) \[ \begin{array}{c}
   \square\square\square\square
   \\\langle\langle\langle\langle
   \alpha
   \end{array} \]
   (c) \[ \begin{array}{c}
   \square\square\square
   \downarrow\downarrow\downarrow
   \langle\langle\langle\langle
   \end{array} \]
   (d) \[ \begin{array}{c}
   \square\square\square\square\downarrow\downarrow
   \alpha\alpha\alpha
   \end{array} \]

4. Write the following as a sum of unit fractions.
   (a) \( \frac{2}{5} \)
   (b) \( \frac{2}{9} \)
   (c) \( \frac{2}{11} \)
   (d) \( \frac{4}{5} \)
   (e) \( \frac{4}{9} \)
   (f) \( \frac{4}{11} \)

5. Write the following as a sum of unit fractions.
   (a) \( \frac{5}{11} \)
   (b) \( \frac{6}{11} \)
   (c) \( \frac{7}{11} \)
   (d) \( \frac{8}{11} \)
   (e) \( \frac{9}{11} \)
   (f) \( \frac{10}{13} \)
6. Use the concept of exponential squaring to find the last two decimal digits of the following numbers.
   (a) \(9^{3328}\)
   (b) \(99^{1541}\)
   (c) \(53^{1541}\)

7. Use the concept of exponential squaring to find the last three decimal digits of the following numbers.
   (a) \(9^{3328}\)
   (b) \(99^{1541}\)
   (c) \(53^{1541}\)

8. For each number below, use the concept of exponential squaring to find the remainder when the number is divides by 91.
   (a) \(9^{3328}\)
   (b) \(99^{1541}\)
   (c) \(53^{1541}\)
Worksheet 5 on primitive counting
Additional Discussion
MATH 4367, Spring 2012

(None of this material requires mathematics beyond the high school curriculum.)

1. Discuss the difference between concepts which are “intrinsic” to mathematics and concepts which are cultural. In your discussion, give examples of both.

2. The Babylonian and Egyptian societies appear to have emphasized drill and rote memorization. (At least that is the material – papyri and tablets – that survive.) A popular concept today suggests that mathematics involves a lot of memorization and use of “magical” formulae. (I use the word “magical” to describe a result or formula which is true but unmotivated, something which is memorized without understanding why it is true.)

Think of results and formulae that you believe need to be memorized, without necessarily understanding their rationale. (The area of a circle? the surface area of a sphere? the Pythagorean theorem? the sine of 30 degrees? trig identities? derivatives or integrals?)

⇒ List as many of these “magical” results as possible! (In your list please give the precise result you are attempting to memorize. For example, if you memorized the Pythagorean theorem as “magical”, please state the theorem, not just its name.)

3. Explain the importance of zero and the difficulty of recognizing its value.

4. Discuss the concept of number as abstract versus concrete. Is “number” a philosophical concept or a concrete object with physical existence?

5. Take a coffee cup or a glass and a ruler and piece of string (or shoe lace) and try to estimate, from the physical evidence, the ratio of the circumference of the glass to its diameter. Is it close to 3? 3\(\frac{1}{2}\)? (Could it be less than 3?) Estimate your answer as carefully as possible, without any modern tools.

Give your answer and a brief description of your reasoning.