Name: ____________________ Date: ____________________
Student ID: ____________________ Section: ____________________

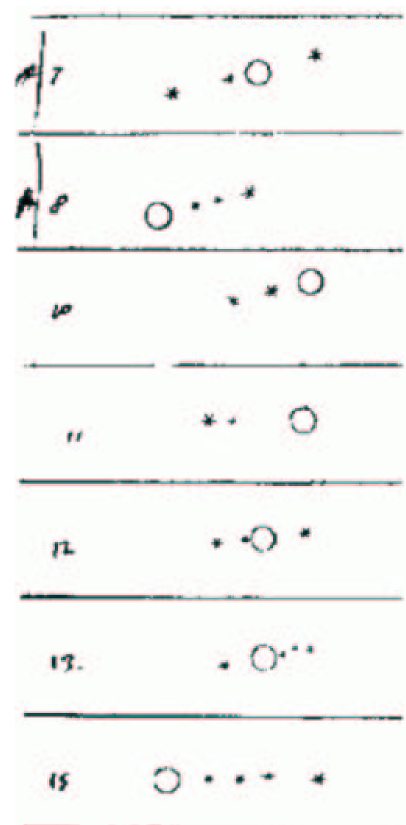
# THE REVOLUTIONS OF THE MOONS OF JUPITER

## I. Objective

In this lab, you will simulate actual measurements of the period and semi-major axis of at least one of the moons of Jupiter. You will then use these values, in addition to Newton's modified version of Kepler's third law, to calculate the mass of Jupiter.

## II. Galileo's Discovery

Back in the early 1600's, Galileo Galilee heard of the recent invention of the telescope and decided to build one himself. In 1609, he started making observations of a number of celestial objects in the sky using his telescope. He made fascinating discoveries which helped change our view of the solar system at the time (as well as create controversy with the Church at the time) by strengthening the Copernican model which suggests a heliocentric, rather than geocentric, based solar system. One of his discoveries was the fact that, when he observed the planet Jupiter, he noticed that it had a number of moons which orbited around it, such as our own Moon orbits around the Earth (a sample of Galileo's observations are shown on the right). Clearly, if moons could orbit around another planet such as Jupiter, then the Ptolemaic model which suggested that all objects orbit Earth could not be correct.



In this lab you will be recreating observations made by Galileo back in the 1600's, except, instead of using a telescope to actually observe the Moons of Jupiter you will be using a computer to simulate the data. The program will show four moons of Jupiter (collectively known as the Galilean moons based on Galileo's discovery) as they orbit around Jupiter. In order of distance from Jupiter, they are: Io, Europa, Ganymede, and Callisto. In this lab you will be plotting the apparent position of at least one of these moons over time relative to Jupiter.

## III. Newton's Modification of Kepler's Third Law

Around the same time, another scientist named Johann Kepler was analyzing detailed observational data of the planets obtained by his predecessor, Tycho Brahe, and determined that planets orbit around the Sun in well defined patterns. His three laws of planetary motion still hold true even to this day. His third law suggests a relation between the average distance of a planet from the Sun (or its semi-major axis), and the amount of time it takes to orbit around the Sun once (or its period).

Many years later, Isaac Newton in his study of the force of gravity, realized that it is the gravitational force between the planets and the Sun which causes them to orbit the way they do. By extension, the orbit of any object around another object (or, to be more correct, the orbit of the two objects around their common center of mass) is due to the gravitational interaction between them. Just as the Moon orbits around the Earth because of the gravitational interaction between them, the Galilean moons orbit around Jupiter for the same reason. Therefore, a variation of Kepler's third law should apply to them as well.

This variation can be stated in terms of a mathematical formula:

$$M_{total} = \frac{a^3}{P^2} \tag{1}$$

where **"a"** represents the average distance between the two objects in question (in astronomical units, which is equal to the average distance between the Earth and the Sun), **"P"** represents the period it takes for one object to orbit once around the other (in years), and **"$M_{total}$"** represents the total mass of the two objects (in solar masses - denoted $M_\odot$, or the mass of the Sun).

For the Earth orbiting the Sun: $M_{total} = (1\ \mathrm{AU})^3/(1\ \mathrm{year})^2 = 1\ M_\odot$, the mass of the Sun.

1. **If the above example yields the total mass of the two objects, why is it equal to the mass of the Sun?**

As a result, if we can determine the average distance of the moons from Jupiter as well as how long each of them takes to orbit, we can measure the mass of Jupiter!

IV. Taking Data

- Open up the program titled **"Revolutions of the Moons of Jupiter"** which can be found under the appropriate course file (the teaching assistant on duty will help you find the program).
- Click on **File** and then click **Log In**. You will be asked to type in your name and the name of your lab partner (no more than two people in a group, please).
- Once you have entered your names, click **OK**.
- Next, under **File**, click on **Preferences** and then click **ID colors**. This will color code the various moons (for easier observations). Once this is on, the moons will be coded as follows: Io (purple), Europa (white), Ganymede (cyan), Callisto (yellow).
- Next, click on **Preferences** and then click **Top View**. This will give you an additional view of the Jupiter system as if you were looking down on it from above (rather than just from Earth - which will appear in the main screen). This window will not pop up until you start running the program.
- Once you have done this, you can click **Run** (under **File**). A screen will pop up which will ask you to set the start time of your observations. Just click **OK** (the default is the current day at midnight).

Now you are ready to start taking observations.

For this exercise, we will be observing a number of moons (your TA will specify exactly which ones). You will see Jupiter located in the center of the screen, and four dots which roughly line up horizontally across the screen (this is the plane in which the moons orbit).

- Find the colored dot corresponding to the moon you are studying and click on it. In the lower, right corner of the screen, you should receive confirmation that you are indeed looking at the correct moon (if not, click on another moon until you find the correct one), as well as a set of coordinates. The important number is the bottom one, which looks like:

  X = (Jup. Diam.)

  This tells you how far away the moon is from Jupiter in terms of Jovian diameters (since by eye you cannot tell how large Jupiter's diameter is, nor how far away the moon is in astronomical units). Included with the number will be the letter "E" or "W". This indicates whether the moon is on the East or West side of Jupiter. **NOTE: This is very important to record!** If you find it difficult to find one of the moons, you may "zoom in" on Jupiter by clicking on one of the four zoom buttons (100X - 400X). As you progress through the lab, if you can't see a particular moon, it may lie outside of your field of view, and you may need to zoom out (lower zoom).
- Once you have found each moon, click on **Record Measurements** and record the distances of the moons from Jupiter in the appropriate box **(do not forget to include "E" or "W"!)**. Also include your measurement in the chart on the last page.
- After you have done this for each moon indicated by your TA, you may continue on to the next night by clicking **Next**.

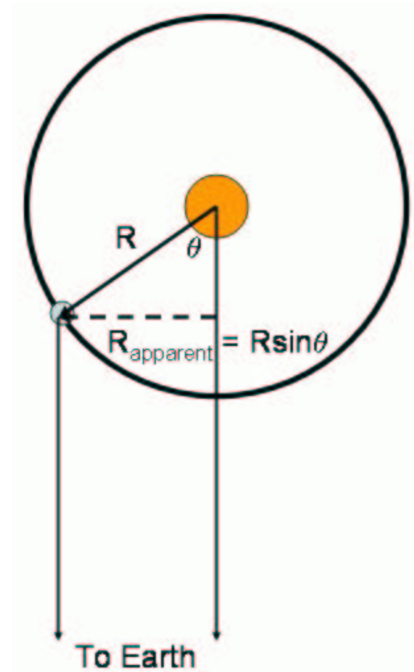
You should take 20 days worth of observations. This does not mean observe for 20 days, because just as in real life, you will have a few "cloudy" days during which you cannot observe Jupiter or the moons. You should record each moon's position 20 times.

- Once you have gathered 20 days of observations, click on **File**, **Data**, and **Analyze**, to analyze your data. A new window will open and will ask you to select a moon for analysis.
- Click on **Select** and then choose one of the moons from the list of moons for which you gathered data. You will see the data you gathered plotted on a graph. The x-axis represents the days over which you observed (plotted in terms of Julian days - a measurement of days) and the y-axis represents the distance of the moon from Jupiter (in Jovian diameters).

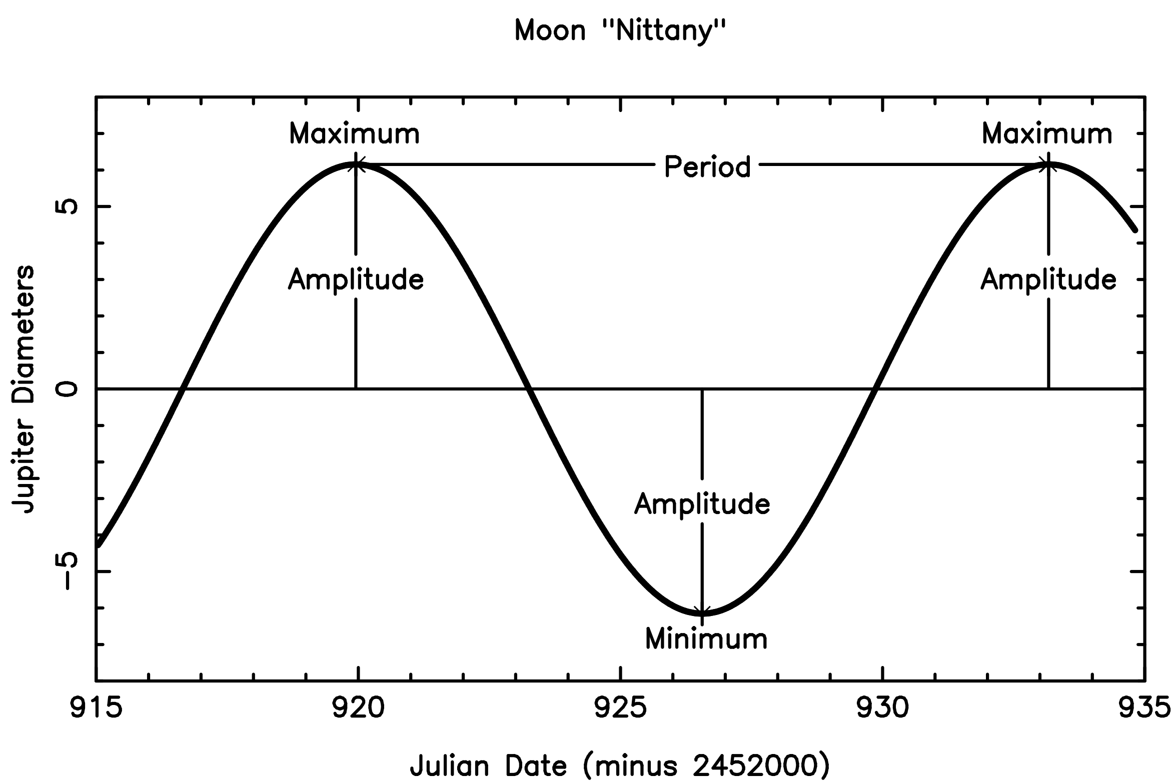
At this point it is important to understand what exactly you are looking at. The graph represents the change in the moon's distance from Jupiter over time, as viewed from Earth. In reality, each moon remains the same distance from Jupiter at all times, but from Earth it appears to move from one side of Jupiter to the other, then back again.

If you plot a moon's projected distance from Jupiter over time, the resultant curve looks just like a sinusoidal (or sine) curve. Sine curves have a few basic properties:

- They are repetitive. Their pattern repeats over time.
- The maximum and minimum values of a sine curve is the same every time (i.e., the peak value is the same for every peak, which is also equal to the trough value). For the moons, this value is equal to its distance from Jupiter.
- The distance between successive peaks is equal to the period of the sine wave. This is the length (of time, in this case) over which the pattern repeats. For the moons, this is equal to the amount of time it takes for them to orbit once around Jupiter.




This is illustrated in the figure below:




Therefore, from this plot it is possible to determine two properties of the moon:

(a) How long it takes to orbit once around Jupiter (its period - P)

(b) How far away it is from Jupiter (its semi-major axis - a)

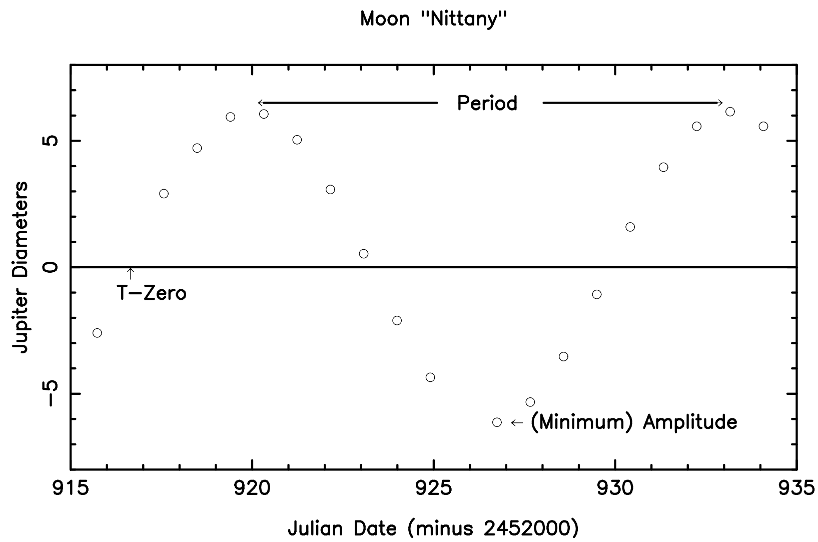
Given these two values, we can use Newton's modified version of Kepler's third law to determine the mass of Jupiter.

V. Determining P and a from your data

If you look at your data points, you should be able to visualize a sine curve passing through them. To plot a sine curve through your points, click on **Plot**, **Fit Sine Curve**, **Set Initial Parameters**. You will be asked to supply an initial guess for a number of parameters, which will be used to plot a fit to the data. The three values are:

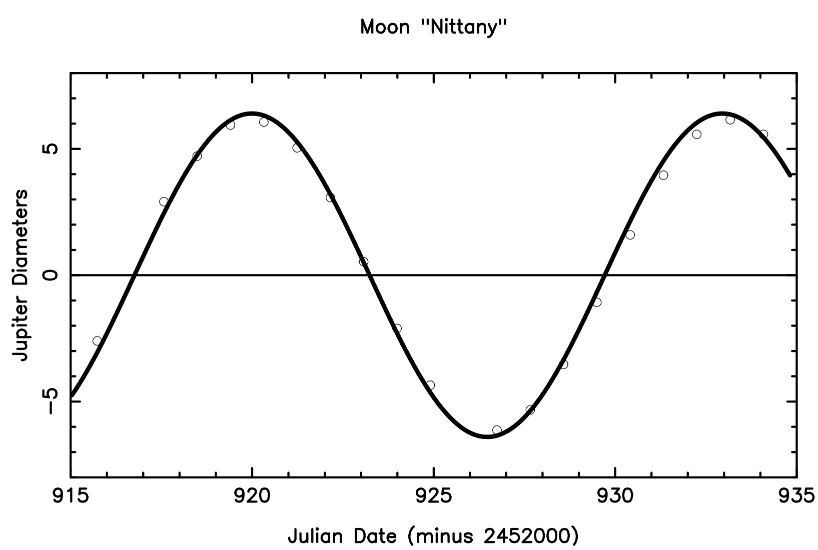
**T-Zero** - the date during which the sine curve crosses the horizontal axis.
**Period** - the distance between successive peaks, in terms of days.
**Amplitude** - the maximum (or minimum) observed distance from Jupiter (in Jovian diameters).

- **To estimate T-Zero**, look at your data points and estimate the date at which your points move from negative (east) to positive (west).
- **To estimate the period**, determine the number of days between peak values.
- **To estimate the amplitude**, look at the maximum and minimum plotted values and use the largest one.



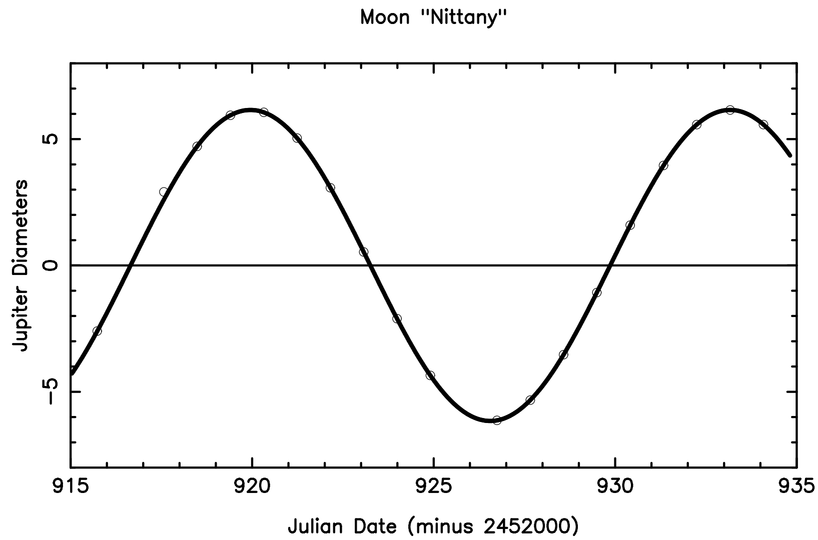

If you click on the screen, it will tell you the position of your cursor.

Once you click **OK**, the program will plot a sine curve based on your initial estimates. The curve may not pass through all the points, but that's okay. You will see three slide bars at the lower left edge of the screen. Use these to adjust the three values so the curve provides a better fit. An **RMS Residual** is listed above these bars. This number tells you how well your curve fits the data. The goal is to make this number as small as possible (note that the RMS Residual is given in scientific notation, and that 1.00E-1 is larger than 9.99E-2).



Adjust the bars one at a time until you get this number as small as you can. If you find that you need to adjust the value of one of your parameters more than the bar will let you, you may need to change your initial guess. Just follow the above instructions to do so.

Once you get the curve to fit the data as best as possible, click Print Current Display under Print (do this once for each member of your group).




VI. Calculating the Mass of Jupiter

Now that you have determined the period and semi-major axis of each moon, you can determine the mass of Jupiter. First, though, we need to convert these values into the appropriate units. There are 365.25 days in a year, and 1050 Jovian Diameters in one AU.

2. **What is the period of each moon, in units of years? (Show your work)**
$P_{\text{Europa}} =$ ______________ $P_{\text{Ganymede}} =$ ______________ $P_{\text{Callisto}} =$ ______________

3. **What is the semi-major axis of each moon, in units of AU? (Show your work)**
$a_{\text{Europa}} =$ ______________ $a_{\text{Ganymede}} =$ ______________ $a_{\text{Callisto}} =$ ______________

4. **Using Newton's modified version of Kepler's third law, what is the mass of Jupiter, as determined by the period and semi-major axis of each moon? (Don't forget to show your work and include the correct units).**
Europa: $M_J =$ ______________ Ganymede: $M_J =$ ______________ Callisto: $M_J =$ ______________

Average: $M_J =$ ______________

**Additional Questions**

5. **There are moons beyond the orbit of Callisto. Will they have larger or smaller periods than Callisto? Why?**

6. **Which do you think would cause a larger error in the mass of Jupiter, a ten percent error in "P" or a ten percent error in "a"? Why?**

7. **The orbit of Earth's moon has a period of 27.3 days and a radius (semi-major axis) of $2.56 \times 10^{-3}$ A.U. (= $3.84 \times 10^{5}$ km). What is the mass of Earth? (Show your work and include the correct units.)**

8. **We did not choose to plot data for Io in order to determine Jupiter's mass for a reason. Why? (Hint: How do the graphs for the different moons compare? How do you think Io's would look?)**

**Data Sheet for the Moons of Jupiter Lab**

| | Date | Europa | Ganymede | Callisto |
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