Calculus of Variation in Wavelet Space

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Outline

- Variational problem in function space.

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- Model evaluation.

Variational method in image procession

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- The restored image $\bar{u}$ is the solution of the variational problem:

$$\bar{u} = \arg\min_{u \in S} E(u).$$

Variation in spatial space (I)

Unconstraint problem

- The energy functional :

$$E(\rho, u) = E_e + E_i = \int_\Omega \rho(|\nabla u|)dx + \frac{\lambda}{2}\int_\Omega (u - u_0)^2 dx,$$

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- Recovered image: $\bar{u} = \arg\min_u E(\rho, u)$.

Variation in spatial space (II)

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- Recovered image:

$$\bar{u} = \arg\min_{u \in B_\sigma} E_e(\rho, u).$$

Steepest descent method

Let $c(p) = \frac{\rho'(p)}{p}$.

Theorem. The anisotropic diffusion equation [Perona-Malik model]

$$\frac{\partial u}{\partial t} = \operatorname{div}\left(c(|\nabla u)\nabla u\right), \quad u(0) = u_0, \quad x \in \Omega$$

$$\frac{\partial u}{\partial \vec{n}} = 0, \quad x \in \partial\Omega$$

provides a steepest descent method for the minimization problem, and the solution $\bar{u}$ is achieved at the stop-time $\bar{t}, i.e., \bar{u} = u(\bar{t})$ with $||\bar{u} - u_0|| = \sigma$.

Remark

The unconstraint problem will lead to the equation:

$$\frac{\partial u}{\partial t} = \operatorname{div}\left(c(|\nabla u|)\nabla u\right) + \lambda(u - u_0), \quad u(0) = u_0, \quad x \in \Omega$$

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Linear model

[in Mumford-Shah's model]

- Energy density: $\rho(p) = \frac{c}{2}p^2$.

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- Leading to isotropic diffusion $u_t = c(\Delta u)$.

Total variation model

[T. Chan, L. Vese, etc.]

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- Energy density: $\rho(p) = p$.
- Conductivity: $c(p) = \frac{1}{p}$.
- Leading to the mean curvature motion

  $$u_t = \operatorname{div}\left(\frac{\nabla u}{|\nabla u|}\right).$$

Edge preserving model (I)

[Perona-Malik]

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- Diffusion equation: $u_t = \operatorname{div}\left(e^{-\frac{|\nabla u|^2}{K^2}}\nabla u\right)$.

Edge preserving model (II)

[Sapiro]

- Energy density: $\rho(p) = \int_0^p xe^{-\frac{x}{K}}dx.$

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- Hard to analyze consistency and stability.

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$$\hat{H}(\Omega) = V \oplus W,\ V \perp W, \quad W = \left(\oplus_{j=0}^{N-1} W_j \right),$$

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- Wavelet decomposition: For $u \in H(\Omega)$,

$$f = T_V f + T_W f = v + w, \quad v \in V, w \in W.$$

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- Across wavelet levels the power spectrum of noise decreases fast.
- At the same level wavelets in diagonal direction are more influenced by noise than those in horizontal and vertical directions.

Variational problem

In Wavelet Space, each wavelet defines a diffusion.

- Energy functional (unconstraint):

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- Energy functional (constraint): E_e with constraint

$$w \in B = \{w; \quad \|w - w_0\|_W \leq \sigma\}.$$

External energy design

Let $\mathbf{w} = [w_1, \cdots, w_n]^T$.

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- The diagonal energy functional on W:

$$\rho(\mathbf{w}) = \sum_{i=1}^{n} g(w_i), \quad g(w_i) = w_i^2 d(w_i).$$

Solution of variational problem

The Euler-Lagrangian equation:

$$\lambda w_i + g'(w_i) = \lambda w_i^0, \quad i = 1, \cdots, n.$$

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- High stability.
- Easy to apply to other wavelet structures (packets, frames, etc.).

Wavelet threshold shrink

[D. Donoho, I. Johnstone] Assume each wavelet coefficient carries a random variable (noise) $\sim N(0, \delta^2)$.

- Hard threshold: (Assume the threshold is γ.)

$$w_i = \begin{cases} 0, & \left|w_i^0\right| \leq \gamma, \\ w_i^0, & \left|w_i^0\right| > \gamma. \end{cases}$$

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- The choice of threshold γ is the most important thing for wavelet shrink. It depends on noise level δ and the dimension of the wavelet space n.

Variational approach to shrink

Assume each wavelet coefficient carries noise $\sim N(0, \delta^2)$.

- Energy function for hard threshold:

$$g(s) = \begin{cases} \delta\,|s| - \frac{\lambda}{2}s^2, & |s| \le \frac{\delta}{\lambda}, \\ \frac{\lambda}{2}\gamma^2, & |s| > \frac{\delta}{\lambda}, \end{cases}$$

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- Solution:

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- The choice of λ determines the accuracy of the models.

Blended variational problem

A non-separate threshold for horizontal and vertical wavelets.

- Assume $\{\psi^h_{j,i}, \psi^v_{j,i}, \psi^d_{j,i}\}_{i=1}^{n_j}$ is the wavelet basis of wavelet subspace W_j:

$$W_j = W_j^h \bigoplus W_j^v \bigoplus W_j^d$$

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$$W_j = W_j^h \oplus W_j^v \oplus W_j^d$$

- Wavelet decomposition: For $w \in W$

$$w = \sum_{j=0}^{N-1} (w^h_{j,i}\psi^h_{j,i} + w^v_{j,i}\psi^v_{j,i} + w^d_{j,i}\psi^d_{j,i})$$

Blended energy

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- Wavelets in a finer space carry more noise than those in a coarser space.
- Wavelets in W_j^d carries more noises than those in $W_j^h \oplus W_j^v$
- Energy functional: Let $m_{j,i} = \sqrt{(w_{j,i}^h)^2 + (w_{j,i}^v)^2}$.

$$E(\mathbf{w}) = \sum \frac{\delta}{2^{j-1}}(m_{j,i} + \frac{\mu \left| w_{j,i}^d \right|}{2}) + \frac{\lambda}{2}\|\mathbf{w} - \mathbf{w_0}\|^2.$$

The solution

Let $\gamma_j = \frac{\delta}{2^{j-1}\lambda}$.

$$w_{j,i}^{h,v} = \begin{cases} 0, & m0_{j,i} \le \gamma_j, \\ w0_{j,i}^{h,v} \frac{m0_{j,i}-\gamma_j}{m0_{j,i}+\gamma_j}, & m0_{j,i} > \gamma_j. \end{cases}$$

$$w_{j,i}^{d} = \begin{cases} 0, & w0_{j,i}^{d} \le \mu\gamma_j, \\ w0_{j,i}^{d} - \operatorname{sgn}(w0_{j,i}^{d})\gamma_j, & w0_{j,i}^{d} > \mu\gamma_j. \end{cases}$$

Other energy functionals

Anisotropic Diffusion 1.

$$g(s) = \delta|s|\left(1 - e^{-\frac{s^2}{2K^2}}\right)$$

$$g'(s) = \delta\,\mathrm{sgn}(s)\left[1 + \left(\frac{s^2}{K^2} - 1\right)e^{-\frac{s^2}{2K^2}}\right]$$

Anisotropic Diffusion 2.

$$g(s) = \delta\,|s|\left(\frac{s^2}{K^2 + s^2}\right)$$

$$g'(s) = \delta\,\mathrm{sgn}(s)\left(1 + \frac{K^2(s^2 - K^2)}{(K^2 + s^2)^2}\right)$$

Energy balance controllers

- The choice of λ.

$$\lambda = \arg\min_{\lambda}(E(\bar{\mathbf{w}}, \delta, n, \lambda)).$$

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- Noise level estimate. (Here 'db3' is used.)

$$\delta = \sqrt{2}\,\mathrm{median}\{|\mathbf{w0}|\},$$

which slightly modifies the estimate given by Donoho and Johnstone:

$$\delta = \mathrm{median}\{|\mathbf{w0}|\}/0.6745.$$

Model Evaluation

- Engineering method: Compare now model to established models.

Model Evaluation

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- Mathematical method: Compare it to the priori estimation (ideal risk, best approximation order, etc.)

Test images: Lena

Figure 1: Lena

Camera Man

Figure 2: Camera Man

Images: Saturn

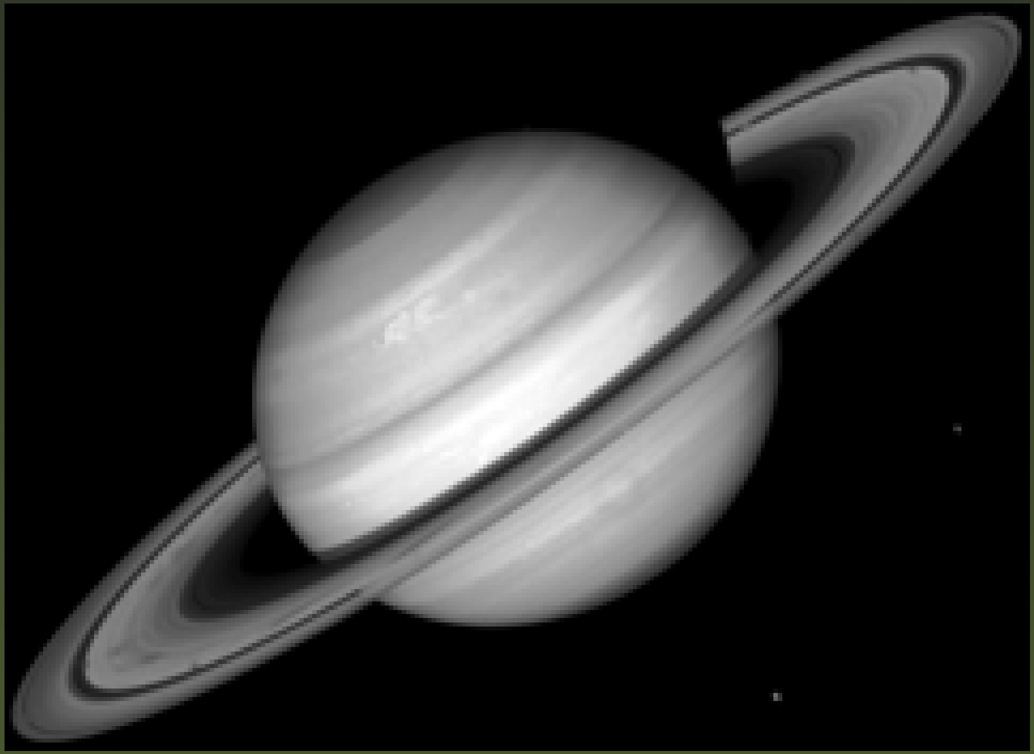

Figure 3: Saturn

Peppers

Figure 4: Peppers

Test Pat

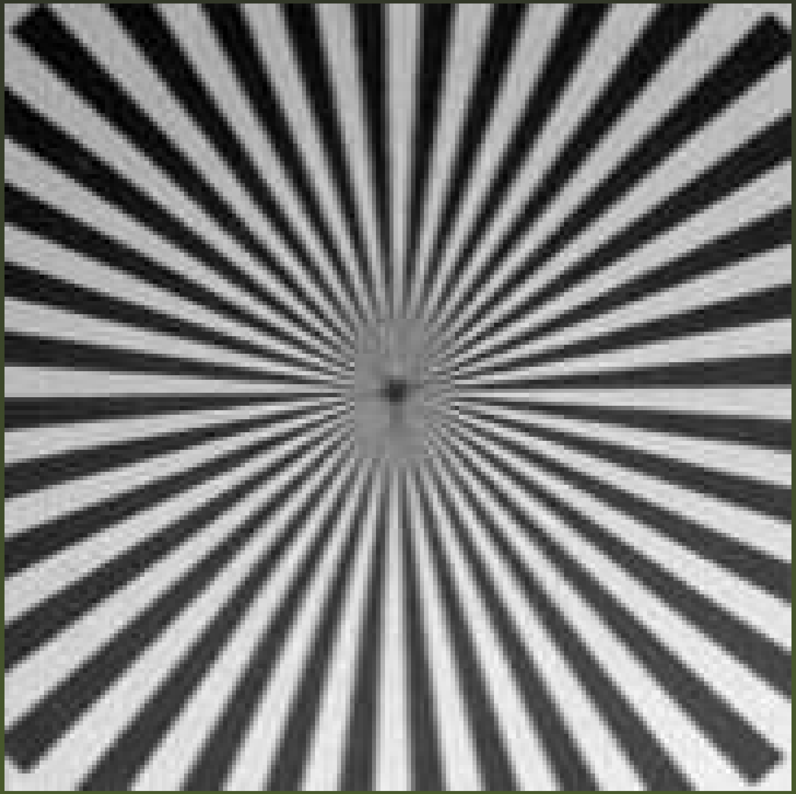

Figure 5: Test Pat

Implementation

- Wavelets: Haar and DB3.
- Methods: soft threshold, hard threshold, blended threshold.
- Test images: Lena, Camera man, Saturn, test pat.
- Noise level: Several.
- Criteria: time and PSNR$=20\log_{10}\left(255/\mathrm{std}(noise)\right)$.

Comparison 1 (Time)

Wavelet: Haar, Noise Std: 15

Images	Lena	Cmrman	Saturn	Pepper	Test pat
Size	49K	66K	144K	132K	66K
Hard	0.190	0.251	0.531	1.121	0.251
Soft	0.191	0.261	0.551	1.152	0.261
VarC	0.210	0.270	0.521	1.161	0.270

$Table\, 1 : CPU\ Time$

Comparison 2

Wavelet: Haar, Noise Std: 5

Images	Lena	Cmrman	Saturn	Pepper	Testpat
Noisy	34.16	34.15	35.37	34.16	34.13
Soft	31.76	30.98	34.98	31.76	28.26
Hard	33.50	34.35	36.91	33.50	32.93
VarC	34.67	34.62	38.28	34.67	32.98

$Table\,7: PSNR$

Comparison 3

Wavelet: DB3, Noise Std: 5

Images	Lena	Cmrman	Saturn	Pepper	Testpat
Noisy	34.12	34.15	35.73	34.16	34.13
Soft	31.49	31.07	37.87	32.87	30.45
Hard	34.31	34.20	39.06	34.19	33.44
VarC	35.18	34.58	39.69	35.17	35.52

$Table\,6: PSNR$

Comparison 4

Wavelet: Haar, Noise Std: 10

Images	Lena	Cmrman	Saturn	Pepper	Testpat
Noisy	28.12	28.26	29.73	28.17	28.15
Soft	26.95	27.43	31.69	29.28	24.05
Hard	29.08	29.80	32.63	30.74	27.24
VarC	30.28	30.49	33.67	31.79	28.59

$Table\, 2: PSNR$

Comparison 5

Wavelet: DB3, Noise Std: 10

Images	Lena	Cmrman	Saturn	Pepper	Testpat
Noisy	28.12	28.26	29.73	28.17	28.15
Soft	28.26	27.70	34.13	30.67	26.97
Hard	30.11	29.77	34.65	31.80	29.34
VarC	31.23	30.65	34.76	32.50	30.27

$Table\,3: PSNR$

Comparison 6

Wavelet: Haar, Noise Std: 20

Images	Lena	Cmrman	Saturn	Pepper	Testpat
Noisy	22.19	22.41	23.76	22.23	22.28
Soft	24.16	24.33	28.27	26.77	20.92
Hard	25.25	25.97	28.52	27.75	22.71
VarC	26.21	26.47	28.50	27.56	23.93

$Table\, 4$: $PSNR$

Comparison 7

Wavelet: DB3, Noise Std: 20

Images	Lena	Cmrman	Saturn	Pepper	Testpat
Noisy	22.19	22.41	23.76	22.23	22.28
Soft	25.71	24.85	29.73	28.27	23.96
Hard	26.52	26.08	29.78	28.89	25.49
VarC	27.01	26.65	29.02	28.14	25.67

$Table\,5\!: PSNR$

Conclusion and Further study

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- The blended energy is suitable for the images with noise of lower level.
- The new method is less sensitive to the chosen wavelet.
- There is a gap between mathematical evaluation and engineering evaluation. This gap perhaps is caused by two reasons:(1) the quantization error; and (2) the idea noise assumption. I guess the first one is the dominant one. Further study is needed.

End

Thank You.