

Non-unique Factorization Properties of Natural Numbers

by

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Abstract: The unique factorization of integers into prime numbers dates back to Euclid and is known today as the Fundamental Theorem of Arithmetic. Recently, the study of arithmetical systems where unique factorization fails has become popular. For instance, in the “Hilbert monoid,”

$$1 + 4\mathbb{N}_0 = \{1, 5, 9, 13, 17, 21, \dots\},$$

unique factorization fails as

$$441 = 21 \cdot 21 = 9 \cdot 49$$

and 9, 21 and 49 cannot be factored in $1 + 4\mathbb{N}_0$. An arithmetic sequence $a + b\mathbb{N}_0 = \{a + bk \mid k \in \mathbb{N}_0\}$ which is also multiplicatively closed is known as an *arithmetical congruence monoid* (or an *ACM*). These, and other simple algebraic systems, have been studied by my REU research groups during the last 8 summers. Several of the theorems I will discuss are a result of these summer investigations. The talk will begin with a general discussion of the current state of undergraduate research in mathematics and the latter part will only presuppose a basic knowledge of congruences and prime numbers.