Math 560
Fall 2005
Homework 8
Assigned Monday, 24 October, 2005

1. (Problem #1, Section 2.5, p. 105-106) Let

$$f(x) = \begin{cases} 1 & x \neq 0 \\ 2 & x = 0 \end{cases}$$

$$g(x) = \begin{cases} 2 & x \neq 1, 2 \\ 3 & x = 1 \\ 4 & x = 2 \end{cases}$$

   (a) Verify that $\lim_{x\to 0} f(x) = 1$, $\lim_{x\to 1} g(x) = 2$, $\lim_{x\to 0} g(f(x)) = 3$, and $g(f(0)) = 4$.

   (b) Do the statements above still hold if

$$f(x) = \begin{cases} x+1 & x \neq 0 \\ 2 & x = 0 \end{cases}$$

2. Let f be defined on $\mathbb{R}^2$. If $\lim_{(x,y)\to(a,b)} f(x,y) = L$ and if the one-dimensional limits $\lim_{x\to a} f(x)$ and $\lim_{y\to b} f(y)$ both exist, prove that

$$\lim_{x\to a}\left(\lim_{y\to b} f(x,y)\right) = \lim_{y\to b}\left(\lim_{x\to a} f(x,y)\right) = L$$

3. Consider the function

$$f(x,y) = \begin{cases} \frac{\sin x - \sin y}{\tan x - \tan y} & \tan x \neq \tan y \\ \cos^3 x & \tan x = \tan y \end{cases}$$

   (a) Determine if $\lim_{x\to 0}(\lim_{y\to 0} f(x,y))$ exists.

   (b) Determine if $\lim_{y\to 0}(\lim_{x\to 0} f(x,y))$ exists.

   (c) Determine if $\lim_{(x,y)\to(0,0)} f(x,y)$ exists.

   (d) Explain the difference between the limits in (a) and (b) and the limit in (c).

4. Discuss the existence of $\lim_{|p|\to\infty} \dfrac{xy - z^2}{x^2 + y^2 + z^2}$