

Math 560  
Fall 2005  
Homework 6  
Assigned Monday, 3 October, 2005

1. (Be able to do this problem, but I will not collect it.) Prove Theorem 4 on page 77. That is, let  $f$  and  $g$  be continuous real-valued functions on a domain  $D$ . Show that the following functions are also continuous:
  - (a)  $f + g$
  - (b)  $fg$
  - (c)  $\alpha f + \beta g$  for  $\alpha, \beta \in \mathbb{R}$
  - (d)  $\frac{f}{g}$  whenever  $g(p) \neq 0$ .
2. (#4, page 80) Let  $f(x, y) = \frac{x^2 y^2}{x^2 + y^2}$  with  $f(0, 0) = 0$ . By checking various sequences, test this for continuity at  $(0, 0)$ . Can you tell whether or not it is continuous there?
3. (#7, page 81) Use the example  $f(x, y) = x^2$  to show that a continuous function does not have to map an open set onto an open set.
4. (#8, page 81) Use the example  $f(x) = \frac{x^2}{1 + x^2}$  to show that a continuous function does not always have to map a closed set to a closed set.
5. (#10, page 81) Show that  $f$  is continuous iff the inverse images of closed sets are closed sets relative to  $D$ .
6. (#12, page 81) How are  $f^{-1}(A \cap B)$  and  $f^{-1}(A \cup B)$  related to  $f^{-1}(A)$  and  $f^{-1}(B)$ ? (Of course you need to prove your claims).
7. If  $f$  is a continuous function from a compact subset  $K \subset \mathbb{R}^n$  into  $\mathbb{R}^m$ , then  $f(K)$  is compact in  $\mathbb{R}^m$ .