Math 560

Fall 2005

Homework 4

Assigned Wednesday, 13 September, 2005

1. (Section 1.6, #7, p. 54) Exhibit a sequence having exactly three limit points. Can a sequence have an infinite number of limit points? no limit points? Could a divergent sequence have exactly one limit point?
2. (Section 1.6, #25, p. 56) If $\{x_n\}$ is a bounded sequence with $\liminf x_n = \limsup x_n$ show that $\{x_n\}$ converges.
3. (Section 1.6, #32, p. 56) Prove that every convergent sequence is Cauchy.
4. (Section 1.6, #34, p. 56) Show that every Cauchy sequence $\{p_n\}$ is bounded.
5. (Section 1.6, #24, p. 56) If $b \leq x_n \leq c$ for all but a finite number of n show that $b \leq \liminf x_n$ and $\limsup x_n \leq c$
6. (Section 1.7, #8 p. 64) Show that the intersection $\cap I_n$ of the nested sequence of intervals $\{I_2\}$ is empty in the following cases:
   (a) $I_n = (0, \frac{1}{n}) \subset \mathbb{R}$
   (b) $I_n = [n, \infty)$.