

Math 560
Fall 2005
Homework 3
Assigned Wednesday, 7 September, 2005

1. (p. 28, #10) If f is a function of two variables, show that if the set of points above the graph of f is a convex set, then f is midpoint convex, i.e. it satisfies the equation:

$$f\left(\frac{1}{2}p + \frac{1}{2}q\right) \leq \frac{1}{2}f(p) + \frac{1}{2}f(q)$$

2. (p. 28, # 14) Given $x_n = 3n + (-1)^n(n - 5) + 7$,
- (a) Find x_1, x_2, \dots, x_{10}
 - (b) Find all the numbers that ever appear twice in the entire sequence.
 - (c) Do any terms appear three times?
3. (p. 28, #15) What is $\bigcup_1^\infty D_n$ where $D_n = \{p \in \mathbb{R}^n \mid |p| \leq n\}$?
4. (p. 28, #16) Show that the collection of all functions defined on a set D , with values in \mathbb{R}^3 is a vector space.
5. Prove that: The intersection of any number of closed sets is closed, but the union of an infinite number of closed sets need not be closed.
6. The interior of a set S is the largest open set that is contained in S .
7. (p. 37, #5) Let $S = \{(x, y) \mid x, y \in \mathbb{Q}\}$
- (a) What is the interior of S ?
 - (b) What is the boundary of S ?
8. (p. 37, #7) Produce an unbounded set with no cluster points.
9. (p. 37 #10) Construct pictures to show that each of the following is false:
- (a) If $A \subset B$ then $\text{bdy}(A) \subset \text{bdy}(B)$
 - (b) $\text{bdy}(S) = \text{bdy}(\overline{S})$
 - (c) $\text{bdy}(S) = \text{bdy}(\text{int}(S))$
 - (d) The interior of S is the same as the interior of the closure of S .
10. (p. 37, #15) Let $A, B \subset \mathbb{R}^2$ be connected sets. Are $A \cap B$ and $A \cup B$ connected?