Math 560
Fall 2005
Homework 2 Partial Solutions
Assigned Monday, 29 August, 2005

1. Prove that $\forall p, q \in \mathbb{R}^n$, $\mid p - q \mid \geq \mid p \mid - \mid q \mid$
2. If $p = (u, v, w)$, show that
   (a) $\mid p \mid \leq \mid u \mid + \mid v \mid + \mid w \mid$
   (b) $\mid u \mid \leq \mid p \mid, \mid v \mid \leq \mid p \mid, \mid w \mid \leq \mid p \mid$
3. Let l be the line determined by the two points p and q. Let $P = \lambda p + (1 - \lambda)q$. Show that if $\lambda > 1$ then $\mid P - p \mid + \mid p - q \mid = \mid P - q \mid$.
4. Show that the intersection of two convex sets is convex, but that the union of convex sets does not have to be convex.

   Solution

   Intersection Suppose that M and L are convex sets. If $M \cap L = \emptyset$ then we are done since this trivially satisfies the definition of convex.

   Pick $a, b \in M \cap L$. Then examine the line segment $\mathfrak{l} = \lambda a + (1 - \lambda)b$ with $0 \leq \lambda \leq 1$. Assume that there is some point c on the line segment $\lambda a + (1 - \lambda)b$ so that $c \notin M \cap L$. Then either $c \notin M$ or $c \notin L$. But since M is convex, $c \in C \ \forall c \in \mathfrak{l}$, and since L is convex, $c \in L \ \forall c \in \mathfrak{l}$, so $c \in M \cap L$, a contradiction. Therefore the intersection of convex sets is convex.

   Union There are many good counterexamples - for one let $M = B(0, 1) \subset \mathbb{R}$ and $L = B(4, 1) \subset \mathbb{R}$. These are both convex, but their union is not.