Math 560
Fall 2005
Homework 10 Partial Solutions
Assigned Monday, 14 November, 2005

1. I won't collect these, but you should be able to do problems number 1, 3, and 8 on pages 134-135.

2. (#5 on page 135) Let a function f be defined in an open set D of the plane, and suppose that f_1 and f_2 are defined and bounded everywhere in D. Show that f is continuous in D.

   Solution

   Assume that f is defined on an open set $D \subset \mathbb{R}^2$, and that f_1 and f_2 are bounded everywhere in D.

   Assume that y is constant. Then f_1 is a function of x, and since that function of x is differentiable, it is continuous. Therefore by the single-variable version of the MVT we have that there is a c so that $\mid f(x_1, y) - f(x_2, y) \mid = f_1(c) \mid p - q \mid = f_1(c) \mid x_1 - x_2 \mid$.

   Similarly, assume that x is constant. Then f_2 is a function of y, and since that function of y is differentiable, it is continuous. Therefore by the single-variable version of the MVT we have that there is a d so that $\mid f(x, y_1) - f(x, y_2) \mid = f_2(d) \mid p - q \mid = f_2(d) \mid y_2 - y_1 \mid$.

   Therefore

$$\begin{aligned} \mid f(x_1, y_1) - f(x_2, y_2) \mid &\leq \mid f(x_1, y) - f(x_2, y) \mid + \mid f(x, y_1) - f(x, y_2) \mid \\ &= f_1(c) \mid x_1 - x_2 \mid + f_2(d) \mid y_2 - y_1 \mid \\ &\leq M \mid x_1 - x_2 \mid + N \mid y_2 - y_1 \mid \end{aligned}$$

   since f_1 and f_2 are bounded. Therefore, as $p \to q$, we have $x_1 \to x_2$ and $y_1 \to y_2$. Therefore, $f(x_1, y_1) \to f(x_2, y_2)$ as desired.

   □

3. (#7, page 135) Let f and g be of class C^1 in a compact set S and let $f = g$ on the boundary of S. Show there must exist a point $p_0 \in S$ so that $\mathbf{D}f(p_0) = \mathbf{D}g(p_0)$.

4. (#9, page 135) Let $f(x, y) = xy$. Show that the direction of the gradient of f is always perpendicular to the level lines of f.