

Math 560
Fall 2005
Homework 1
Assigned Wednesday, 24 August, 2005

1. Complete exercise 19 on page 11: Derive the following from the axioms of the real field:

- (a) For any $a, b \in \mathbb{R}$, if $a < b$ then $-b < -a$
- (b) For any $x \in \mathbb{R}$ $x^2 \geq 0$ with equality only if $x = 0$
- (c) For any $x, y \in \mathbb{R}$, if $x^2 + y^2 = 0$ then $x = y = 0$.

2. Complete exercise 20 on page 11:

- (a) If $0 < m < n$ then $m^2 < n^2$ and $\frac{1}{n} < \frac{1}{m}$
- (b) If $m < n < 0$ show that $m^2 > n^2$ and $\frac{1}{m} > \frac{1}{n}$

3. Let E be a nonempty subset of an ordered set; suppose α is a lower bound of E and β is an upper bound of E . Prove that $\alpha \leq \beta$.

4. Let A be a nonempty set of real numbers which is bounded below. Let $-A$ be the set of all numbers $-x$ where $x \in A$. Prove that $\inf(A) = -\sup(-A)$.

5. Let $\beta \in \mathbb{R}$, $p, q, p_1, q_1 \in \mathbb{R}^n$. Then

$$(\beta p) \cdot q = \beta(p \cdot q) = \beta(q \cdot p)$$

and

$$p \cdot (q_1 + q_2) = p \cdot q_1 + p \cdot q_2$$

6. You should be able to do problems 6-10, 12-13 in the Exercises on pages 10-11. I will not collect these.

7. You should also be able to do the following problem, which I will not collect: Find the sup and inf of each of the following sets of real numbers:

- (a) All real numbers of the form $2^{-p} + 3^{-q} + 5^{-r}$ where $p, q, r \in \mathbb{Z}^+$.
- (b) $S = \{x \mid 3x^2 - 10x + 3 < 0\}$
- (c) $S = \{x \mid (x - a)(x - b)(x - c)(x - d) < 0\}$ where $a < b < c < d$.