Math 477 - Homework
Fall 2006

Problems about Cauchy's Theorem.

88. Read Section 2.8. This, by the way, is not a presentable problem. But you should make sure that you read the section.
89. Prove or disprove: A group of order 35 is cyclic.
90. Prove or disprove: There exists a nonabelian group of order 21.
91. Prove or disprove: A group of order 42 has a nontrivial normal subgroup.
92. Prove or disprove: Any two nonabelian groups of order 21 are isomorphic.

Problems about direct products of groups.

Before reading this definition, look back at problem 84.

Definition 10 *If $G_1, G_2, \ldots, G_n$ are n groups, then their (external) direct product $G_1 \times G_2 \times \cdots \times G_n$ is the set of all ordered $n-$tupls $(a_1, a_2, \ldots, a_n)$ where $a_i \in G_i$ for $i = 1, 2, 3, \ldots, n$, and where the product is component-wise "multiplication" in the product on G_i.*

93. If G_1 and G_2 are groups, Prove or disprove: $G_1 \times G_2 \simeq G_2 \times G_1$.
94. If G_1 and G_2 are cyclic groups of order m and n, respectively, Prove or disprove: $G_1 \times G_2$ is cyclic iff m and n are relatively prime.
95. Let G be a group and $A = G \times G$. Let $T = \{(g, g) \mid g \in G\} \subset A$.
    (a) Prove or disprove: $T \simeq G$
    (b) Prove or disprove: $T \triangleleft A$ iff G is abelian.

Problems about abelian groups.

96. Prove the **Fundamental Theorem of Finite Abelian Groups**: A finite abelian group is the direct product of cyclic groups.
97. Prove or disprove: There are 10 nonisomorphic abelian groups of order 144.
98. How many nonisomorphic groups of order 60 are there? Prove your claim.