

Math 477 - Homework
Fall 2006

Problems about sets.

1. Prove or disprove: If $A \subset B$ then $A \cup C \subset B \cup C$ for any set C .
2. Prove or disprove:
 - (a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
 - (b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
3. If $C \subset S$, let $C' = S - C$. Prove the de Morgan Rules for subsets, i.e.
 - (a) $(A \cap B)' = A' \cup B'$
 - (b) $(A \cup B)' = A' \cap B'$
4. Let S be a set. For any two subsets of S we define

$$A + B = (A - B) \cup (B - A) \text{ and } A \cdot B = A \cap B$$

Prove that

- (a) $A + B = B + A$
 - (b) $A + \emptyset = A$
 - (c) $A \cdot A = A$
 - (d) $A + A = \emptyset$
 - (e) $A + (B + C) = (A + B) + C$
 - (f) If $A + B = A + C$ then $B = C$
 - (g) $A \cdot (B + C) = A \cdot B + A \cdot C$
5. If C is a finite set, let $m(C)$ denote the number of elements in C . If A and B are finite sets, prove that
$$m(A \cup B) = m(A) + m(B) - m(A \cap B)$$
 6. For finite sets A and B show that $m(A \times B) = m(A)m(B)$.
 7. Prove or disprove: A set which has n elements has 2^n subsets.