```latex
%\documentclass{beamer}

\documentclass[handout]{beamer}
\usepackage{pgfpages}
\pgfpagesuselayout{resize to}[letterpaper,landscape,border shrink=10mm]


\usetheme{singapore}
\usepackage{amsfonts,amsmath, amsthm, amssymb, latexsym, epsfig}
\usepackage{hyperref}

\usepackage[all]{xy}
\usepackage{xspace}

\usepackage[mathscr]{eucal}

\usepackage[english]{babel}


%\usepackage{hyperref,amsmath}
%\usetheme{default}
%\usetheme{AnnArbor}%blue and yellow
%\usetheme{Antibes}
%\usetheme{boadilla}
%\usetheme{Bergen}
%\usetheme{berkeley}%ok
%\usetheme{Berlin} %i like this
%\usetheme{cambridgeus}  %red
%\usetheme{copenhagen} %ok
%\usetheme{darmstadt}
%\usetheme{dresden} % I like this, too
%\usetheme{frankfurt} %fine
%\usetheme{goettingen} %notes on right
%\usetheme{hannover} %notes on left
%\usetheme{ilmanau} %not found
%\usetheme{juanlespins} %lots of notes on the top
%\usetheme{luebeck} %nice
%\usetheme{madrid} %ok - like many others
%\usetheme{malmoe} %nice and clean - just black and blue
%\usetheme{marburg} %pretty - notes on right
%\usetheme{montpellier} %also nice and clean
%\usetheme{paloalto} %lots of blue
%\usetheme{pittsburgh} %very clean - titles on right
%\usetheme{rochester}
%\usetheme{szeged} %nice and simple, but I don't like the footer
%\usetheme{warsaw} %i like this, too
```

```
%\usetheme{boxes}

\newtheorem{thm}{Theorem}
%\renewtheorem{lemma}[thm]{Lemma}
\newtheorem{cor}[thm]{Corollary}
\newtheorem{proposition}[thm]{Proposition}
\theoremstyle{definition}
\newtheorem{defn}[thm]{Definition}
\newtheorem*{rem}{Remark}
\newtheorem*{ex}{Example}
\newtheorem*{question}{Question}
\newtheorem*{conjecture}{Conjecture}
\newtheorem*{results}{Known Results}


\begin{document}

\title{Canonical genus and the Whitehead doubles of pretzel knots}
\author{Mark Brittenham\inst{1} and Jacqueline Jensen\inst{2} }
\institute[University of Nebraska - Lincoln and Sam Houston State University]{
\inst{1}
Department of Mathematics\\
University of Nebraska - Lincoln\\
\
\\
  \inst{2}%
  Department of Mathematics and Statistics\\
  Sam Houston State University}
  \date{ Joint Mathematics Meetings\\
  7 January 2007 }

\begin{frame}
\titlepage
\end{frame}

\section[Definitions]{Defintions and Background}
\subsection[Definitions]{Definitions}
\begin{frame}
\frametitle{Definitions}
\begin{defn}
$K \subset S^3$ is a \alert{knot} if it is a smooth $1-$manifold embedded in $S^3$.
\end{defn}

\vfill

\begin{defn}A \alert{Seifert surface} of a knot $K$ is a compact, connected,
```

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orientable surface $S \subset S^3$ such that $\delta S=K$.
\end{defn}

\vfill
\begin{defn}The \alert{genus} of a knot $K$, denoted $g(K)$ is the minimal genus
of all Seifert surfaces with boundary $K$.
\end{defn}

\vfill

\end{frame}

\begin{frame}
\frametitle{Definitions, cont.}

\begin{defn}A Seifert surface is \alert{canonical} if it is obtained from a diagram of
$K$ by applying Seifert's algorithm.
\end{defn}

\begin{ex}
%Example
\scalebox{.10}{\includegraphics{6_1_oriented.pdf}}
\scalebox{.10}{\includegraphics{6_1_sc.pdf}}
\scalebox{.10}{\includegraphics{6_1_circles.pdf}}
\scalebox{.10}{\includegraphics{6_1_surface.pdf}}
\\
\tiny{Example from http://www.sgwater.org/math/knots/, referenced 12/27}
\end{ex}
\end{frame}

\begin{frame}
\frametitle{Definitions, cont.}

\begin{defn}
The \alert{canonical genus} for $K$, $g_C(K)$ is the minimal genus among all
canonical Seifert surfaces of $K$.
\end{defn}

\begin{fact}
Both genus and canonical genus are hard to calculate.
\end{fact}



\end{frame}
```

```latex
\section[Question]{Question}
\begin{frame}
\frametitle{A Question}

\begin{center}
\Large How are genus and canonical genus related?
\end{center}


\begin{fact}
$g(K) \leq g_C(K)$
\end{fact}


\begin{thm}[Gabai, 1986, and others] Seifert's algorithm gives a minimal genus
Seifert surface when the algorithm is applied to an alternating projection of an
alternating knot $K$.
\end{thm}

\begin{cor}
For alternating knots, $g_C(K) = g(K)$.
\end{cor}


\end{frame}

\section[Known Results]{Known Results}

\begin{frame}
\frametitle{Ways to Compute Genus}

\begin{itemize}

\item (Gabai, 1984)  With a minimal genus surface candidate, sutured manifold
theory can verify its minimality.

\vfill

\item (Morton, 1986) The $z-$degree of the HOMFLY polynomial, $P_K(v,z)$ of a
knot $K$ is at most twice the canonical genus, i.e., $\textrm{max deg}_z P_K(v,z)
\leq 2g_C(K)$.\\
\
\\
A canonical surface whose genus is half of the $z-$degree must therefore have
genus equal to canonical genus.  This condition cannot always be met, giving
examples where the inequality is strict.
```

\end{itemize}

Both methods succeed for alternating knots. The second method has computed genera for knots up through 12 crossings (Stoimenow, 2002).
\vfill

\end{frame}

\begin{frame}
\frametitle{Known Results}
\begin{itemize}
\item H. R. Morton (1986) -- A twisted Whitehead double of the trefoil knot has the canonical genus at least 3, while its genus is 1

\vfill

\item A. Kawauchi (1994) -- There exists a knot K so that $g_C(K) - g(K) = 2n$ for any $n \in \mathbb{N}$.

\vfill

\item J. J. Tripp (2002) -- The canonical genus of a twisted Whitehead double of a $(2,n)-$torus knot is n, i.e. it is equal to the crossing number of the original knot.

\vfill

\end{itemize}

\end{frame}
\section[More Definitions]{More Definitions}

\begin{frame}
\frametitle{More Definitions}
\begin{defn}[Whitehead double]
Let J be the Whitehead clasp in an unknotted solid torus, $S^1 \times B^2$. Let $h: S^1 \times B^2 \to S^3$ be an embedding taking $S^1 \times\{0\}$ to a knot K. The knot $h(J)$ is the Whitehead double of K with n twists, where n is the linking number of the longitude of the torus with K.
\end{defn}

\begin{fact}
If $n=w(P)$ for the projection P of K, there is a standard projection of the Whitehead double which appears untwisted. This is the Whitehead double that we study.

```
\end{fact}

\begin{center}
\scalebox{.12}{\includegraphics{trefoil.pdf} \includegraphics
{trefoil_double_closeup.pdf}}
\end{center}

\end{frame}


\section{The Conjecture}
\begin{frame}
\frametitle{The Conjecture}
\begin{conjecture}[Tripp, 2002]
The crossing number of a knot is equal to the canonical genus of its Whitehead
double.
\end{conjecture}

\vfill

\begin{results}
\begin{itemize}
\item Tripp (2002) -- true for $(2,n)-$torus knots
\item Nakamura (2004) -- true for $2-$bridge knots
\end{itemize}
\end{results}

\end{frame}

\section{Their Methods}

\begin{frame}
\frametitle{Morton's Inequality}

\begin{thm}[Morton, 1986] The $z-$degree of the HOMFLY polynomial $P_K(v,z)$
of a knot $K$ is at most twice the canonical genus, i.e.
$$\textrm{max deg}_z P_K(v,z) \leq 2 g_C(K)$$
\end{thm}

Tripp and Nakamura show that the $z-$degree of the HOMFLY polynomial of the
double is $2n=2c(K)$ for the appropriate knot.
\end{frame}


\section{Our Results}
```

```
\begin{frame}
\frametitle{Definition}

\begin{defn}
$P(k_1, k_2, \ldots k_n)$ is a pretzel link if it is the sum of tangles $k_1, k_2, \ldots,
k_n$ with $k_i \geq 1$.  It is a knot iff either $n$ is odd or $n$ is even and exactly
one $k_i$ is even.
\end{defn}

\begin{ex}
\begin{center}
The $(-2,3,7)$ pretzel knot.

\scalebox{.15}{\includegraphics{pretzel.pdf}}
\end{center}
\end{ex}

\end{frame}


\begin{frame}
\frametitle{Our Results}

\begin{thm}[B-J, 2006]
If $K$ is a pretzel knot $P(k_1, \ldots, k_n)$ with $k_1, \ldots, k_n \geq 1$ then
$g_C(W(K))= k_1+\cdots +k_n = c(K)$.
\end{thm}
\
\\

Proof is similar to the techniques of Tripp and Nakamura.

\end{frame}

\section[References]{References}


\begin{frame}
\frametitle{References}
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\end{itemize}
\end{frame}
\end{document}