ART AND MATHEMATICS

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Abstract. In this paper we are going to explore how mathematical concepts have been used in famous artwork. In doing so, we will analyze different artists from different ages and some of the techniques they used for their work. We will then discuss how mathematical concepts can be seen as artistic work, and how different artistic concepts can be seen as mathematical ones.

1. Introduction

The purpose of this paper is to provide a connection between art and mathematics. More precisely, we want to provide a way to view art as a mathematical creation, and we also want to view mathematical concepts as artistic developments. To start we want to provide the following two definitions:

Art: A nonscientific branch of learning; one of the liberal arts.

Mathematics: A science (or group of related sciences) dealing with the logic of quantity, shape and arrangement.

and propose the following definition:

Mathematical Art: A nonscientific science dealing with the abstract logic of quantity, shape and arrangement.

2. Art as a Mathematical endeavour

In this section we discuss some of the different techniques that artist have used, and we make a connection between those techniques and mathematics. The techniques that we are going to explore are:

- One Point Perspective
- Pointillism
- Contour Lines
- Cubism
- Abstract Art

FIGURE 1. Leonardo Da Vinci's *Last Supper*

2.1. **One-point Perspective.**

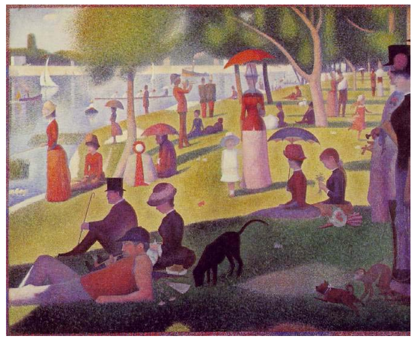
Let us begin with the one-point perspective. To define it, let us say that it is a perspective in which all parallel lines converge on a single point on the horizon or the eye level line, and we will call it the focus point. The one-point perspective serves two purposes. One is that it helps the artist place objects in a nice proportional manner according to that point. And the second purpose is that a greater emphasis is placed on that object that is located at the focus point. For example, in Leonardo Da Vinci's (1452-1519) *The Last Supper*, Figure 1, the focus point is somewhere around Jesus's forehead.

Notice that this artwork is huge. In fact, it is 15 by 29 feet in dimensions. It is interesting to know that Leonardo created this artwork by placing a nail in the center of the wall and radiating strings outward from this nail to place the twelve apostles in a proportional way to Jesus. By proportional we mean that the men seem to be of equal sizes relative to each other. Also note that the second puporse is accomplished by letting the walls, the roof, or ceiling, and the table converge to that focus point in Jesus's forehead. This makes Jesus the focus of the artwork. Of course, one of the reasons the emphasis is placed on him is because of the religious significance he has in Christianity.

2.2. **Pointillism.**

Now instead of using just one point, pointillism is an artistic technique that systematically applies color theory in a scientific way through the use of colored points. One purpose of this technique is to enhance the artwork, or in other words, make the artwork brighter. The reason pointillism accomplishes this purpose is that in the process of applying color through the use of points, there are many gaps in between all of

FIGURE 2. George Seurat's *Sunday Afternoon on Isle De La Grande Jatte*



these points. So throughout the entire artwork there are many white gaps (since the paper or canvas is white) that the human eye interprets as light radiating from the drawing. Because of this, it seems as if the piece of art has light emitting from it and thus making it brighter. A good example of pointillism, is George Seurat's (1859-1891) *Sunday Afternoon on Isle de La Grande Jatte*, Figure 2.

In this painting, you can see the brightness of the sun that is depicted by the refraction of light in the grass and the water. Another thing to point out is that pointillism also enhances the artwork by making it more detailed. This means that the objects are more sharp or distinguishable from each other. For example, you can tell that the dog in the lower center of the painting is a dog and not part of the shadow casted on the grassy floor. Another way to think of pointillism is that it is similar to a computer screen. The more pixels (points) that you have the sharper the image is, thus giving you a detailed representation of the object it wants to depict. The *Evening Breeze*, Figure 3, by Henri-Edmond Cross (1856-1910) gives a more obvious representation of pointillism.

You can tell from the drawing that somewhat larger colored points were used in this artwork. Not only that, but the actual sunlight itself is being depicted as coming through the ship's sails.

2.3. **Contour Lines.**

Moving away from a set of colored points, we now talk about a set of points that make up a line (or a curve) that in turn defines the edges of shapes. These are what we call contour lines. They draw a rough sketch of an object, whether it be an animal or an inannimate object.

Figure 3. Henri-Edmond Cross' *Evening Breeze*

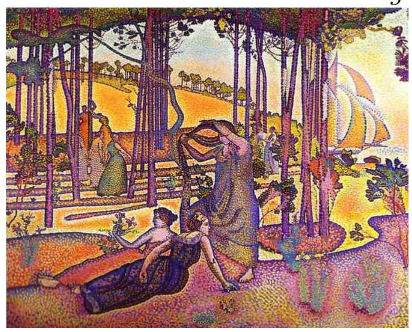


Figure 4. Pablo Picasso's *The Dog*

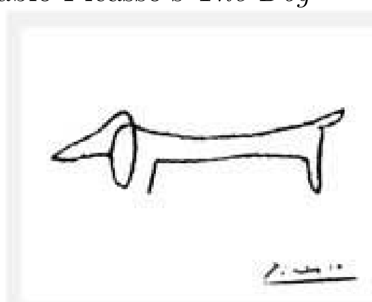
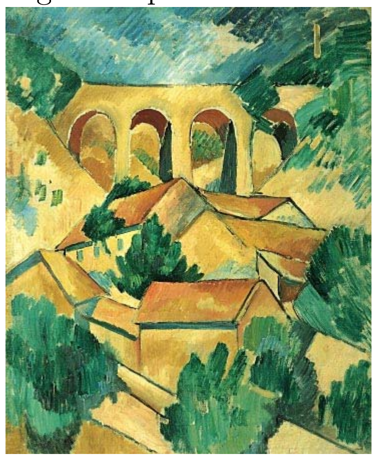


Figure 5. Pablo Picasso's *The Camel*

Two pieces of artwork that give a good representation of contour lines are *The Dog*, Figure 4, and *The Camel*, Figure 5, both of which were drawn by Pablo Picasso (1881-1973).

FIGURE 6. Georges Braque's *Viaduct at L'Estaque*



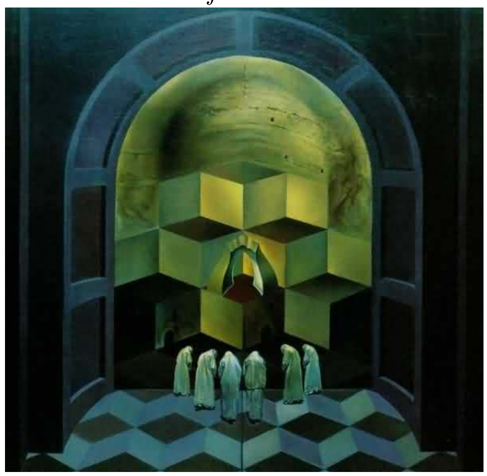
One should note that Pablo Picasso is not world-famous because of these two simple drawings, but instead because of his abstract drawings and painting. It is also true that the technique of contour lines does not necessarily have to limit itself to the use of just one line (or curve).

2.4. **Cubism.**

The technique of cubism is the reduction and fragmentation of natural forms into abstract, often geometric structures usually rendered as a set of discrete planes. Another way to think of cubism is that the artist sees his or her objects as being composed of simple shapes. For example, if the artist wanted to draw a person he or she will depict the head simply as an oval and the rest of the body as being created from rectanges that define the different body parts. While observing Georges Braque's (1882-1963) *Viaduct at L'Estaque*, Figure 6, one can see how the rooftops of the houses in the drawing are portrayed by simply making them rectangles. Also, the aquaduct in the backgroung does not have much detail but it is obvious what it is. An artistic reason for the lack of detail is that objects that are closer to the observer seem to have more detail than objects that are far away. And it makes sense in real life. One sees things that are closer to one much better than objects that are far away.

The artwork in Figure 6, by Georges Braque, was created in the early stages of the artistic movement that used the technique of cubism. A latter work under this movement that we demonstrate is the *Skull of Zarbaran*, Figure 7, that was painted by Salvador Dali (1904-1989).

FIGURE 7. Dali's *Skull of Zurbaran*



In this work, one can see the use of discrete planes used to create the cubes that serve as a basis to draw the skull and also the temple which the monks are entering. Note that the floor is also composed of cubes also. Thus, through the use of a simple geometric figure (cube), Dali creates an intrinsic artwork, meaning that the two objects or ideas of the painting being depicted seem to fit together, without one oversimplifying the other.

2.5. **Abstract Art.**

Abstract art can be defined as art that departs significantly from natural appearances. This means that the artwork seems to be a representation of real life, with the exception of one or more laws or rules of logic being ignored. For example, let us say that we have a picture of a deer in the woods, but it has human feet. Now this would be considered abstract art because there is no way that deer have human feet. To provide a glimpse of abstract art, one can again look at Salvador Dali's artwork, namely *The Persistence of Memory*, Figure 8.

In this painting everything seems to be a nice depiction of Spain's ocean view, but the problem is that the clocks are melting. If one applies enough heat to any object, it will definitely melt, but the weird feeling one gets from this painting is that if, for example, one is at the same place where the clocks are melting, most likely one will also be melting.

Another artist that created abstract art is Maurits Cornelis Escher (1898-1972). His artwork is famous for his use of mathematical concepts

FIGURE 8. Dali's *The Persistence of Memory*

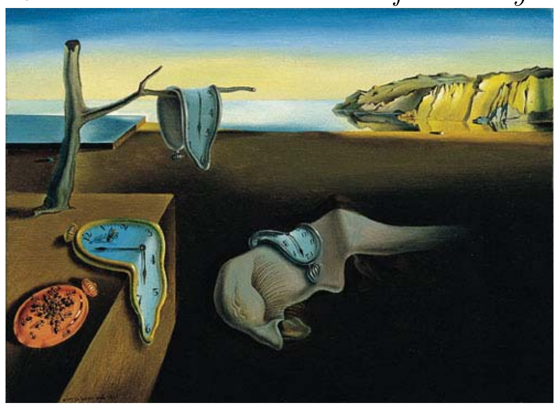
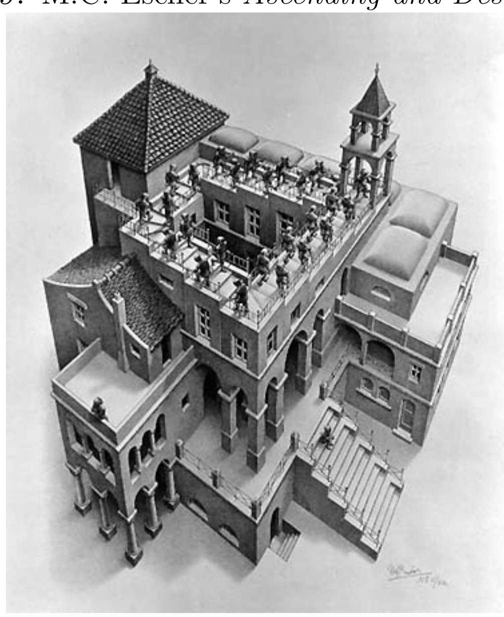


FIGURE 9. M.C. Escher's *Ascending and Descending*



to develop his work. His artwork tests the laws of logic to the extreme. For example, the logic behind the action of moving up the stairs and getting to a higher place is defied by Escher's work as it can be seen in *Ascending and Descending*, Figure 9.

As one can see, the men in the rooftop seem to be going up the steps, but at the end they end up at the same place where they started. So one may ask the question, "How can that be?" The architecture of the

FIGURE 10. M.C. Escher's *Waterfall*

building seems to be perfect. However, the roof is also increasing in elevation if you follow it clockwise.

Another example drawing from M.C. Escher that defies basically the same rule of logic is the *Waterfall*, Figure 10. This time, if one follows the path of the water counterclockwise after it has fallen, one ends up at the top of the waterfall. Gravity is being defied here because water will not be pumped to a higher elevation without the use of some mechanical device that gives it a push. Also, one can see Escher's use of the polyhedra that lie on top of the columns to distract the observer from the simple path of the water. These are just a few examples of how M.C. Escher departs significantly from natural appearances.

2.6. **Mathematical Connection.**

Now, the connection that we propose exists between the techniques artist have used and mathematics is that they follow essentially the same pattern as the development of Euclidean geometry. The Euclidean axioms that we refer to are:

- **Point:** A straight line can be drawn between any two points.
- **Line:** A finite line can be extended infinitely in both directions.
- **Plane:** A circle can be drawn with any center and any radius.
- **Space:** Given a line and a point not on the line, only one line can be drawn through the point parallel to the line.

Thus, the one-point perspective, or focus point, can be compared to the axiom in Euclidean geometry that defines a point. Then, pointillism is just a multiple application of that axiom that defines a single point. Moving to the next axiom in Euclidean geometry, we develop the countour line, which roughly speaking is an ordered existence of infinitely many points between any two points. Now that we have lines defined, we can develop planes to create cubes which we can compare to the artistic technique of cubism (Note we have used the axiomatic definitions in a loose way). The next step in Eucliden geometry after developing three dimensional space, is to obscure it, or to play with it. This means that three dimensional space exists, but we want to see what happens when a rule of logic is ignored. With this we get abstract art or mathematical art. Therefore, through the use of different artistic techniques, one can see that this development of art can be viewed as a mathematical creation.

3. Mathematics as an Artistic Endeavour

In this section we want to demonstrate how mathematics can be seen as an artistic endeavor. We approach this by showing how art is a helpful tool in mathematics and that without it interpreting even simple mathematical concepts can be tiresome.

3.1. **Graphing.**

We fist start by mentioning that in mathematics, we use various different graphing techniques to represent all sorts of functions. For example, we use cartesian coordinates to graph basic functions such as the sine and cosine function from trigonometry in the Cartesian coordinate plane, Figure 11.

We also use polar coordinates to graph other functions that are parametarized differently, Figure 12.

There are many other types of graphs that we use in mathematics, which also includes the complex-plane to graph complex functions, Figure 13.

But the point that we want to make is that instead of looking at those numerical values, in a table for instance, mathematicians use graphs to represent that exact information in a way that is more readable. In other words, through the use of a graph, an artistic tool, one can

FIGURE 11. Cartesian Coordinate Plane

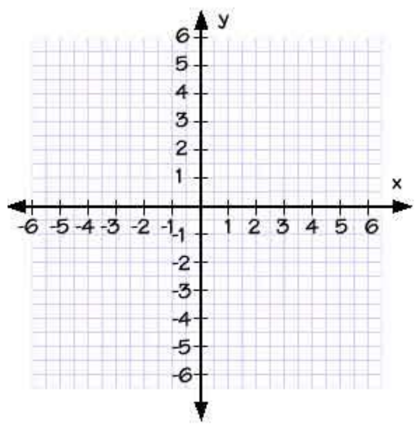
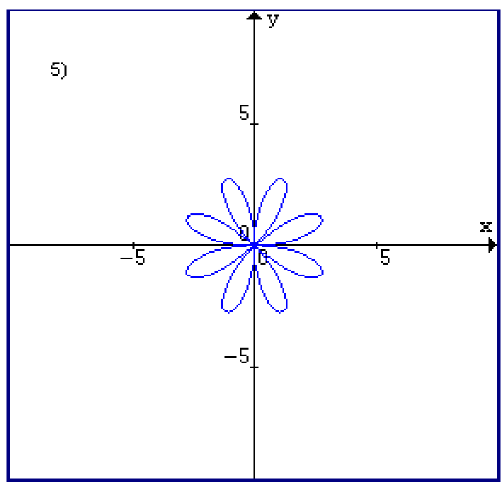


FIGURE 12. Polar Coordinate Plane (r=4sin(4θ))



interpret, analyze and even make conclusions from the information or data being depicted by the graph.

3.2. **Statistics.**

Another specific example, statisticians use graphs to interpret or make inferences about the information given to them. For example, they use scatter plots to graph the data in a way that helps them determine the conclusions they can make from whatever experiment they are working on.

Figure 13. Complex Identity Function Graph

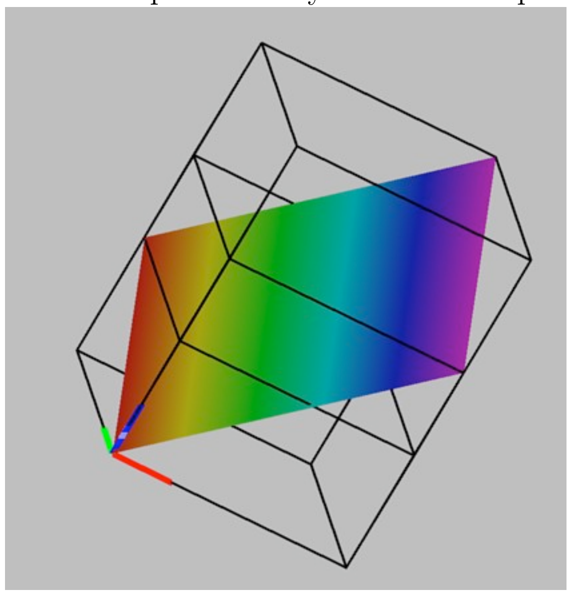
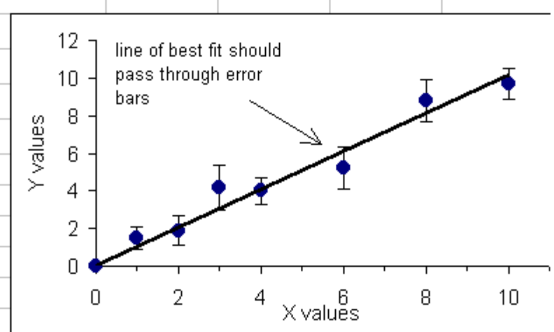


Figure 14. Graphical Representation of Numerical Data

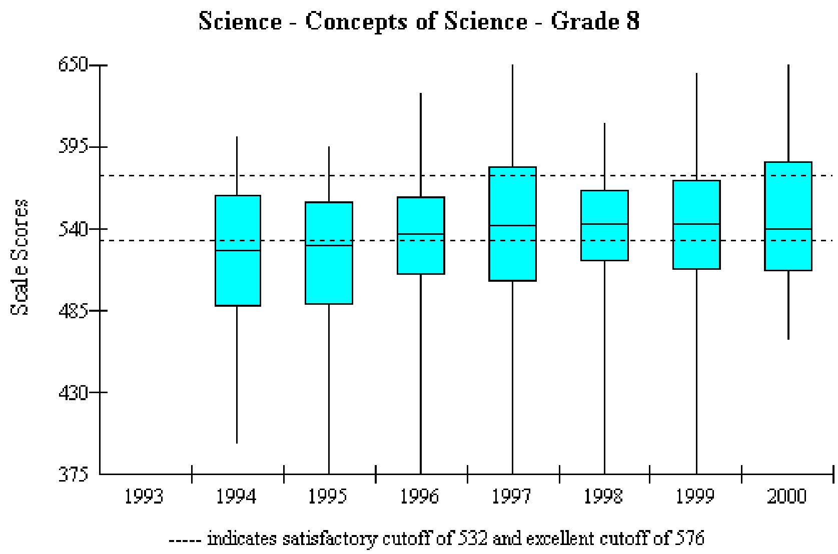
x	y	CI
0	0	0
1	1.5	0.6
2	1.9	0.8
3	4.2	1.2
4	4	0.7
6	5.2	1.1
8	8.8	1.1
10	9.7	0.8



From the diagram in Figure 14, one can see that it is much easier to say that the data follows a linear distribution from the graph than from the list of numbers in the columns to the left of the graph. Another useful graphing technique (artistic technique) that statistician rely on is box and whisker plots.

The graph in Figure 15 is actually a multiple representation of a box and whisker plot, which demonstrate how multiple information can be plotted in a single graph in order to compare it all at once. A reason

FIGURE 15. Box-and-Whisker Plot



comparison box-and-whisker plots are used for is that statisticians can compare highs, lows, medians, and quartiles all in one graph. With these examples, one can see how statisticians, through the use of different graphing techniques, can be seen as artists in their field of work.

3.3. **Geometry.**

Another field in mathematics that relies heavily in artistic concepts to develop itself is geometry. One of the topics that is covered in geometry is tessellations. Tessellations are the arrangement of geometric shapes to create a pattern. For example, in Figure 16 we see that squares and triangles are arranged so that there is a continuous pattern throughout.

To make it even more artistic, one can actually give some color in a certain way to produce a more obvious pattern. Tessellations can also be produced in other spaces, such as hyperbolic space. Figure 17 gives an example of a tessellation with color added to it, and Figure 18 gives an example of a hyperbolic tessellation with color applied to it.

One should realize that tessellations are present in almost every place one goes to. For example, the tiling of the floor, even though squares

FIGURE 16. Tessellation

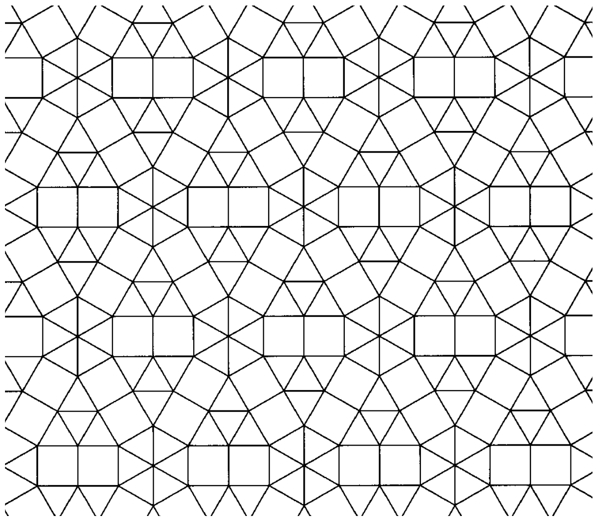
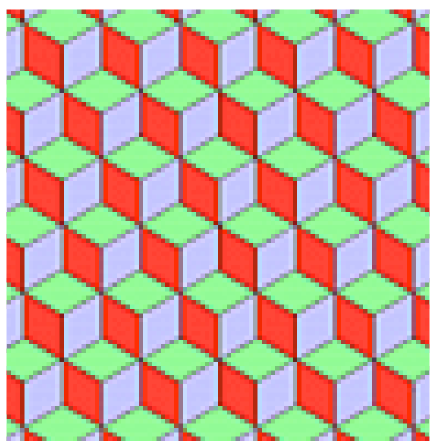


FIGURE 17. Tessellation with color



might be the only geometric shape that is used, is a tessellation. The roof in houses, generally have a pattern that appears to be a tessellation because the shingles are placed in a repeating pattern throughout the roof. Tessellations, or patterns, can occurr in the office at work, as can be seen in Figure 19. Not only can one see the tessellation occuring in the tiling behind the flower bases in the glass cases, but one can also see the symmetry in the entire room by the location of the glass cases.

Figure 18. Hyperbolic Tessellation

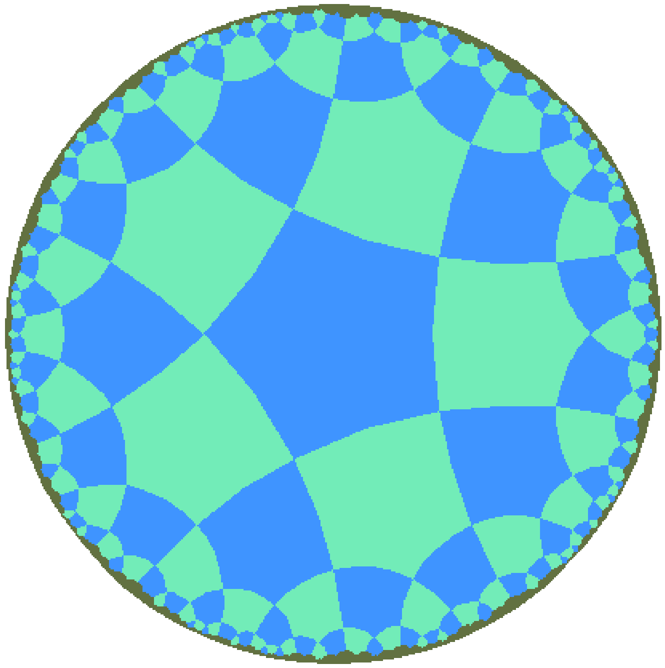


Figure 19. Symmetry in the Office Place

From the discussion of graphing with different coordinate systems, graphing different data plots, and the discussion of geometry, one can see how art serves not only as a tool to describe mathematical concepts but also that mathematics in itself is an artistic endeavour.

Thus, we conclude by saying that art and mathematics play a complementary role to each other when viewed in the way we have presented them in this paper. We want to leave with the following quote:

Let us help one another to see things better –Claude Monet

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