Picture Perfect

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Abstract

Images are used everywhere, but not all images are as good as they could be. People then turn to Photoshop to correct images. There are very specific algorithms that are used to correct these images to make them clearer. The focus of this paper will be on simple correction techniques for image restoration, image reconstruction, and image retouching.

1 Applications

There are many different applications that use algorithms and formulas in order to correct images. Sharpening out-of-focus photographs may be currently the most popular use, due to the increasing number of people using Photoshop or other image correction programs.

Another application is generating images from medical scanners. Medical scanners such as x-ray machines, CAT Scan machines and MRI machines use algorithms to take the collected data and make them into an image. With so many advances in these different types of techniques we are able to make clearer and more accurate images for doctors and other medical professionals to diagnose from. These techniques have also help make great leaps for the 3-D medical scans.

Image correction techniques also help provide evidence for law enforcement. Law enforcement uses these techniques to clear up images from security cameras in order to see a license plate number or someone's face. This way they can give the public more accurate information to help find criminals.

2 Breaking Down An Image

There are many different techniques that are used to correct images. Some are complicated while others have a simpler approach. However, all image correction techniques must get their data from the image by using a simple break down. The breaking down is the only thing that is the same throughout all of the different techniques. Some of the techniques use different variables or may require a few more details about the image, but they all have the same basic break down, which goes as follows:

- Labeling Each Pixel
- Define the Brightness Intensity
- Finding Distortion
- Finding Error

These are going to be described in detail in the following four sections.

2.1 Labeling Each Pixel

Since the images that most people work with now a days are digital we will have to first understand what a pixel is. A pixel is an abstract solid color dot or square that when put in a group of other pixels will form an image. When an image is zoomed in on we can see what is called pixelization. This is were we can see the pixels clearer so this then make the image looks distorted.

To keep an image organized we need to label the pixels within the image. Each image is made up of $N \times N$ pixels which will be labeled by (i, j) which is the i^{th} column and the j^{th} row, and $1 \leq i, j \leq N$ (Yu).

2.2 Brightness Intensity

There are two main color scales for images. Either grayscale or color images are used to gather data. Grayscale images are made up of 64 different shades of gray. While color images are made up of at least 256 different shades of color. This makes grayscale images usually easier to work with, only because they have a smaller color range. You may also convert color images into grayscale images by a method that involves decreasing the number of colors that you would like to allow in your image. We do this by using the numbers that each color has (you find these numbers by a program that can read the colors) and putting it into color categories. When converting to grayscale we usually use the following color breakdown for f_{ij} which is how we define our brightness intensity.

$$\text{Black: } 0 \leq f_{ij} \leq 63$$
$$\text{Dark Gray: } 64 \leq f_{ij} \leq 127$$
$$\text{Light Gray: } 128 \leq f_{ij} \leq 191$$
$$\text{White: } f_{ij} \geq 192$$

NOTE: $f_{ij} \geq 0$ (Yu)

2.3 Finding Distortion

Distortion is also known as lost data. Data is usually lost around the edges during transfer, but it can be lost at anytime. Lost data is just rows or columns of pixels that have be cut off the image. We call this our distortion **d** where:

$$\mathbf{d} = (d_{ij}; i, j = 1, ..., M; M \leq N)$$

So is the new number of rows or columns that we have after the data is lost around the edges.

Now that we know what **d** is we have to apply it to the brightness intensity **f**. We will do this by a known transformation T which represents the blurring process (Yu). So this is going to give us Model 1 which is:

$$\mathbf{d} = T(f)$$

2.3.1 Transformation *T*

Transformation T is very difficult to find because it is arbitrary, so depends on the specific application that it is being used for. In order to find this transformation the first step is to identify what must be done to the image. Then, by taking an existing transformation of which the function is already known adjustments can be made to make the transformation work for the particular image.

2.4 Finding Error

The error in the image is also called the noise of an image. The noise of an image is considered to be the objects that appear in the image that are not actually there. Noise is sometimes caused by the reflection of light off of a surface in the image onto the lens of the camera. We will call the noise, or error, **e**. We then need to add this to Model 1 so that we can get the total distortion. So the new equation, or Model 2, looks like:

$$\mathbf{d} = T(f) + e$$

NOTE: **f** and **e** are assumed to be $N(0, \sigma^2)$ (Yu).

3 Classic Methods

There are many different methods to chose from to perform different image operations. However, over the past few years people have made great strides in making these techniques better. The classic methods or some variation of them still exists today.

Most of the classic restoration methods were viewed as a system of linear equations with M^2 equations and N^2 unknowns. Some of the favored classic restoration methods are Inverse Filtering, Wiener Filtering, Least-Squares Solutions with Regularization, and Psedoinverse Solution. These methods are the bases for most current methods. However, these methods have problems that constantly need to be corrected. These problems can include:

- Noise Amplification -

  This is when the noise is increased or made more prominate within the image.

- Produced Oscillations Around Sharp Changes in Intensity -

  This is when there is an extreme change in color, from the blackest black to the whitest white, extra lines or circles can appear around or near the change.

So in order to avoid having these problems another method, Regularized Restoration Method, must be done on top of the other method that was chosen. This method may have to be used more then once in order to correct the image. So then when using classic restoration methods, one method is chosen and then the Regularized Restoration Method is used several times. This can very confusing and complicated.

4 The Bayesian Approach

The Bayesian Approach fits perfectly with the image break down, so there is no changing of variables or rearranging of equations. The Bayesian Approach was demonstrated by Philip Yu in his paper A Simple Statistical Project: Image Reconstruction. The Bayesian Approach has two main steps to insure that it works correctly.

- The first step is setting a suitable distribution for the image **f** and a similar distribution for the noise function **e**.

- The second step is to maximize the posterior probability of the image **d** using Bayes's Theorem given the image **f** under constraints.

4.1 Suitable Distribution

The first part of the Bayesian Approach is to set a suitable distribution for the image **f** and the noise **e**. In order to set a suitable distribution take Model 2 and distribute a blurring function to image **f**.

$$\mathbf{d} = \mathbf{f} \times \mathbf{h} + \mathbf{e}$$

The blurring function will be know as $\mathbf{h} = (h_{uv}; (u, v = -w, ..., w))$ where $w =$ the width of blurring. This then gives the equation:

$$d_{ij} = \sum_{p=i-w}^{i+w} \sum_{q=j-w}^{j+w} f_{pq} h_{i-p,j-q} + e_{ij}$$

$$\mathbf{i}, \mathbf{j} = 1, ..., M$$

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Now in order to get the conditional probability of total disturbance d_{ij} given the image f_{ij}

$$Pr(d_{ij}|f_{ij}) = \frac{1}{\sqrt{2\pi}\sigma} exp\left\{ -\frac{1}{2\sigma^2} \left(d_{ij} - \sum_{p=i-w}^{i+w} \sum_{q=j-w}^{j+w} f_{pq} h_{i-p,j-q} \right)^2 \right\}$$

When this equation is reduced it will look like the following approximation.

$$\propto exp\left\{ -\frac{1}{2\chi^2(f)} \right\}$$

where

$$\chi^2(f) = \frac{1}{\sigma^2} \sum_{i=1}^{M} \sum_{j=1}^{M} \left(d_{ij} - \sum_{p=i-w}^{i+w} \sum_{q=j-w}^{j+w} f_{ij} h_{i-p,j-q} \right)^2$$

when reduced looks like

$$\chi^2(f) = \frac{1}{\sigma^2} \sum_{i=1}^{M} \sum_{j=1}^{M} e_{ij}^2$$

These equations are going to take the sum of all the columns and the sum of all the rows and average them together, and then when it is placed into a computer program it will apply it to the image.

4.2 Setting Constraints

In order to set constraints there must first be an understanding of where the brightness comes from in an image. Brightness comes from the emission of grains that have an energy λ in each pixel of the image. The grain's energy λ is usually seen as a typical shade of color which is read and then interpreded as the pixel's brightness. There are n_{ij} grains in each pixel (i, j) and

$$n_T = \sum_{i=1}^{N} \sum_{j=1}^{M} n_{ij}$$

will be constant by the conservation of energy because we are dealing with energy from the image data. The brightness intensity is now defined as $f_{ij} = \lambda n_{ij}(i, j = 1, ..., N)$ therefore,

$$T = \sum_{i=1}^{N} \sum_{j=1}^{N} f_{ij}$$

So then,

$$T = \lambda n_T$$

is a constant.

So our prior probability of the image **f** is

$$Pr(f) \propto \frac{n_T!}{\prod_{i=1}^{N} \prod_{j=1}^{N} nij!}$$

and by using a combination of Bayes's Theorem and the approximation from before to find the posterior probability will give the following equation.

$$S(f) - \frac{\lambda}{2}\chi^2(f)$$

This equation will now be maximized and will be subject to the constraint

$$T = \sum_{i=1}^{N}\sum_{j=1}^{N} f_{ij}$$

because λ is an undetermined constant (Yu). This is then inputted into a computer and this will calculate the constraints that for the distribution and will output the final corrected image.

5 Conclusion

Images are not always "perfect" when they are taken. However, there are many methods that are used to correct an image. This process, The Bayesian Approach, along with a few more algorithms and a computer that is programed will change the image tremendously and make it a better image for the viewer to look at.

References

[1] Yu, Philip L. H. "A Simple Statistical Project: Image Reconstruction", American Statistical Association, 1994.