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Evolution of Mathematics 467

April 30, 2009

Arabic Contributions to Mathematics

Abstract: My paper is about recounting the contributions to the advancement of mathematics by Arabic people from the seventh to eleventh century. I explain some of the early history of the Arab empire and make known the contributions of leaders such as Haroun al-Rashid and his son al-Mamun. I also illuminate the mathematical contributions of the Arab mathematicians al-Khwarizmi, al-Khayyam, and Alhazen.

1. Introduction

Many of history's most accomplished mathematicians and contributors to mathematics come from the Muslim empire. This was not always a known and accepted fact. There were leaders like Haroun al-Rashid and his son Abu Jafar Abdullah al-Mamun who helped to pave the way for the advancement of mathematics by transforming Baghdad into a center of learning where intellectuals from all over could come and study. There were scientists like Mohamed al-Khwarizmi who traveled to the city and worked on the advancement of mathematics so they could better perform scientific explorations and advance in their scientific fields. There were also translators like Hunayn ibn Ishaq who came to Baghdad to translate great works from the Greeks to Syriac, Alhazen who advanced Geometry and number theory, and men like Omar ibn Ibrahim al-Khayyam who studied and wrote books in mathematics and even poetry. Telling the history of the formation of the Muslim empire gives insight into how Muslim people were prepared to advance mathematics. Also, understanding the contributions of al-Mamun, al-Khwarizmi, al-Khayyam, and Alhazen, provides proof of the many contributions of Arabic people to the advancement of mathematics over hundreds of years.

2. History

Major advances in mathematics that had been occurring in cities in Europe like Rome ended around 529 A.D. when the eastern emperor Justinian decided the pagan philosophical schools of learning were threatening Christianity. After this, significant mathematical advances began again in Persia where many intellectuals went after the closing of the other schools (Boyer 193). A new era of mathematical advancement began again in Persia years later after the time of Mohammed (Morgan 53).

Arabic or Islamic history is heavily influenced by different groups of people with a diverse “vision for the political, theological, and intellectual development of the state” (Morgan 53). The Mohammedan era begins in 622 A.D., when the prophet Mohammed fled Mecca, where he had been preaching for around ten years, due to a threat to his life. Mohammed was a religious and military leader who began the “expansion of the Islamic state” (Boyer 225-226). One of the places they conquered was Alexandria in 641 A.D., which was the “mathematical center of the world” (Boyer 226). There was much fighting between Arab conquerors until 750 A.D. that split the people into eastern and western Arabs. The eastern Arabs were led by Caliph al-Mansur who founded the city of Baghdad (Boyer 225-226). This began the Abbasid rule in Baghdad until 945 A.D. (Morgan VI).

In A.D. 813, Caliph Abu Jafar Abdullah al-Mamun became the leader of the Muslim empire. The Muslim empire included areas like Spain, Iraq, Persia, Byzantine

Egypt, North Africa, Turkmenistan, and Afghanistan (Morgan VI-VII). Prior to his reign, in A.D. 790, under the rule of al-Mamun's father, Haroun al-Rashid, the budding city of Baghdad had already been made a center for learning for philosophy, science, and literature (Morgan VII). Baghdad had the "first major public and private libraries since the time of Rome and Alexandria" (Morgan 59). Two of the libraries were said to be so large that one of the owners would need 140 camels to transport all of the books and the other would need 400 camels for transportation (Morgan 59).

Al-Mamun was fascinated by the early Greek and Roman historical centers of learning, and by historical figures. He wished to continue the growth of Baghdad and turn it into the next Alexandria or Rome. He did this by building the House of Wisdom or "Bait al-hikma" and inviting the worlds best thinkers and translators such as Mohamed al-Khwarizmi, the Banu Musa brothers, Hunayn ibn Ishaq, and al-Kindi to come study in his city (Morgan 56-57). He had strong beliefs in inclusion and was willing to "absorb peoples, histories, secrets, and behaviors, and let them flourish there" (Morgan 50). There is a story told about al-Mamun that exemplifies his quest for knowledge. When he defeated the Byzantine Empire, al-Mamun wanted as his offering a copy of the *Almagest*, written by Ptolemy on astronomy, to be added to his House of Wisdom instead of any gold or other wealth (Morgan 56). Also, in A.D. 829, Al-Mamun added two observatories in Iraq so his astronomers could "not only read the hidden message of space but to better map and document it" (Morgan 57).

A way in which scientific discovery in the House of Wisdom is different then discoveries of the past is that the Arabic scientific research was not "fit in a preconceived notion of the universe dictated by theology". They instead tried to "understand the complexity of creation" which opened new doors for intellectual discussion and examination (Morgan 60). Al-Mamun was very open to having his city's scientists broaden their understanding of presently known facts and even have them disproved as opposed to those in the past like Pythagoreans who wanted to shape the world to fit into their mold.

There were different political and religious groups with views on how the empire should be run. Al-Mamun believed in rationalism and intellectual elitism. He believed that "if the most gifted of men are not allowed to think freely, they cannot create or invent, and so they cannot fulfill the will of God" (Morgan 56). One of these groups were the Mutazilites, religious thinkers that Al-Mamun allied himself with for support. Their outlook at the time was radical because they believed "the Qur'an is created, not His eternal Word" and that "reason is the highest expression of God" (Morgan 51).

In opposition to Al-Mamun were the traditionalists, some of whom believed in anti-intellectualism. They thought Al-Mamun's beliefs were "departing from the intent of the Prophet" (Morgan 51). Their leader was Ahmad ibn Hanbal and al-Mamun put him in jail, which actually helped the traditionalists gain support (Morgan 54). Al-Mamun's successor Caliph al-Mutawakkil was a traditionalist, and he overthrew the Mutazilites (Morgan 63). It is easy to see how the political and religious groups of the time could hinder and discourage advancement.

3. Mohammed ibn Musa al-Khwarizmi

Mohammed ibn Musa al-Khwarizmi is considered the Father of Algebra (Morgan 56). He was born around 780 A.D. in Khiva (Morgan 85). Some Arabs knew him as al-Majusi, which means magician. This makes some think his original religion was

Zoroastrianism. He also knew ancient Hebrew, so some believe he may have been Jewish (Morgan 86).

Al-Khwarizmi left home to travel to the House of Wisdom in 832 A.D. after being asked to come by al-Mamun. Al-Mamun wanted him to "search for God in the numeral" (Morgan 86). Studying at the House of Wisdom granted him access to a large amount of scientific and mathematical materials, and it allowed him to collaborate with many others of like mind for the first time (Morgan 86-87). The author Morgan, describes the House of Wisdom as "the world's first think tank" (Morgan 88).

Something al-Khwarizmi worked on was tracking down a copy of a Hindu text by Brahmagupta called Brahma Sphuta Siddhanta or Opening of the Universe which the Arabs called Sindhind. This book was about astronomy, predicting eclipses, and calculating planetary position (Morgan 87). It is from this book he learned of the character zero representing "nothingness" which is "the Hindu character shaped as a dot, a pinpoint of blackness like a negative star" (Morgan 88).

Al-Khwarizmi wrote a booklet about zero. He explained that zero had different functions by saying, "When after subtracting, nothing is left, write a small circle, lest the place remain empty. The small circle must occupy the place, lest there will be fewer places and, for instance, the second is taken for the first" (Bunt 227). This was al-Khwarizmi's explanation on what zero is and how zero should be used. This booklet shows that Arab mathematicians not only used zero, but they also used placeholders.

Al-Khwarizmi also wrote a book in 820 A.D. called al-kitab al-muhtasar fi hisab al-jabr w'al-muqabala (Smoller). This is translated into The Compendious Book on Calculation by Completion and Balancing (Smoller). Al-jabr means "restoring" as in subtraction in an equation from one side to the other, and al-muqabala means "comparing" as in subtracting from both sides the same amount (Morgan 92). There were six types of equations, linear and quadratic, that al-Khwarizmi found solutions for in this book (Smoller). The mathematics in this book is not written how mathematics is today. Everything is written out in words. An explanation of a problem by al-Khwarizmi is written like this example.

> "If the instance be, ten and thing to be multiplied by thing less ten then this is the same as if it were said thing and ten by thing less ten. You say therefore, thing multiplied by thing is a square positive; and ten by things positive; and minus ten by things is ten things negative. You now remove the positive by the negative, then there only remains a square. Minus ten multiplied by ten is a hundred, to be subtracted from the square. This, therefore, altogether, is a square less a hundred direms" (Smoller).

This example is saying that "ten and thing to be multiplied by thing less ten" or (10 + x)(x - 10) can be solved by first multiplying "thing" by "thing" or (x)(x) to get "a square positive" or x^2 (Smoller). Then multiply positive and negative 10 by "thing" to get "ten things negative" or –10x and "ten things positive" or +10x which cancel each other out (Smoller). Last "minus ten" is multiplied by ten to get the above x^2 – 100 as the solution to the problem (Smoller).

Another mathematics problem al-Khwarizmi worked on was a problem originally introduced by Heron. "Given an isosceles triangle with base 12 and legs each equal to 10, inscribe a square inside the triangle with one side along the base and the other two vertices on the legs. What is the side of the square?" (Boyer 233).

Al-Khwarizmi solved this problem by first using the Pythagorean theorem to show a^2 + 6^2 = 10^2 which calculates the altitude of the triangle as a = 8. Then the area of the isosceles triangle and the area of the inscribed square are calculated to be 48 and x^2 respectively. Finding the area of the isosceles triangle will help in finding the value of the side of length x of the square. The area of the bottom right and bottom left triangles are equal, so together they equal the heights of length x multiplied by the bases of length $y = 6 – (x/2)$. The area of the top triangle is one half of the base x multiplied by the height $(a – x)$ which equals $(8 – x)$. The area of these four inscribed figures equals the area of the isosceles triangle which is $x^2 + x(6 – (x/2)) + (x/2)(8 – x) = 48$. Solving this equation shows that $x = 4.8$ which is the length of the side of the square.

Mathematics at this time could be done in three ways: finger-counting, Arabic characters, and the Hindu method. Finger counting was mostly useful for small things in business, and the Arabic characters in mathematics were very complicated. The Hindu method used the decimal system of numbers from zero to nine, and al-Khwarizmi saw how this system could make calculations easier, so it was then made useful to some Arabs (Morgan 89). Al-Khwarizmi helped introduce these concepts into Arabic mathematics, which gave mathematicians after him the tools for “detaching the source of mathematics from the physical and moving it into the purely abstract” (Morgan 92). He wrote about this in a book called Al-Khwarizmi on the Hindu Art of Reckoning and copies of this book later spread this knowledge to medieval Europe as well (Smoller).

4. Omar ibn Ibrahim al-Khayyam

Omar ibn Ibrahim al-Khayyam is one Arab man that was able to benefit from the work of al-Khwarizmi about two hundred years later. Omar al-Khayyam was a poet, philosopher, astronomer, and mathematician who lived from 1044 to 1123 A.D (Kasir 1). A story of al-Khayyam is that he and two of his friends made a pact that whoever became wealthiest first would help the other two be successful (Kasir 1). Due to this pact made with his friend Nizam ul Mulk, the grand yizir to the Selijukian sultan, al-Khayyam received an annual allowance to support him while he concentrated his time in his fields of study (Kasir 2). Then around 1079, al-Khayyam went to Merv to be the royal astronomer of the court. He led the committee to better calendars, and the calendar they made was in "error of one day in about every 5000 years" (Kasir 2 and 5).

Only three of the ten books al-Khayyam wrote have survived. The three that survived are The Rubaiyat, Demonstrations of Algebraic Problems, and Some Difficulties of Euclid's Definitions. His other seven books were about metaphysics, government, natural science, and the calendar he worked on in Merv (Kasir 5).

Al-Khayyam was mainly a scientist, and, because of this, his mathematical discoveries were not his main focus in his writing. Following Arab convention, his books only explored mathematics when it was directly needed for understanding problems from science, surveying, commercial transactions, and inheritance law (Kasir 2).

Of al-Khayyam's three surviving books, The Rubaiyat was his book of poetry. It was originally translated into English by Edward Fitzgerald, and was first published in 1851 (James 9). It is obviously not a direct translation, but most of the original ideas are preserved, as well as the rhyme scheme where the first, second, and fourth line of each stanza rhyme (James 15). Al-Khayyam's poetry was viewed "by orthodox Mohammedans as heretical, materialistic, and even atheistic" (Kasir 3). His writing put him in danger of persecution from religious extremists (Kasir 3). In a biography written about al-Khayyam by al-Zeizani, he thinks that al-Khayyam was "restrain[ing] his tongue and pen" and therefore holding back on some of his more radical views to stay out of danger. He also claims the only reason al-Khayyam participated in the pilgrimage to Mecca was either to hide his real ideas about religion or just curiosity (Kasir 3).

Omar al-Khayyam's other surviving book, Demonstration of Algebraic Problems, is split into 10 chapters with six main topics: definitions of "fundamental notions of algebra," tables of simple and compound equations, first and second degree equations, cubic equations, fractional equations, and "remarks on the work of Abu'l Jud" (Kasir 21).

The meaning of the word algebra is "the art of completion and reduction" and al-Khayyam defines it in his book as "the science that aims at the determination of numerical and geometrical unknowns" (Kasir 21). He believed that algebra could only be proven or verified by using geometry to prove the algebra. He proved this true for all the known equations of degree less than three, but he could not use this method of proof for cubic equations (Kasir 21-22). As a rule, he applied "geometry to his algebraic solutions, and algebra to the geometric constructions of his equations" (Kasir 22).

Omar al-Khayyam's greatest contribution to mathematics was through his work on cubic equations. Al-Khayyam "gave a complete classification of the forms of cubic equations and constructed a geometrical solution for each type" (Kasir 29). However, he did solve cubic equations by intersecting a parabola with a circle (Omar). He was not the first mathematician to try this route, but he was able to make generalizations that could be used for all cubics that earlier mathematicians could not. Binomial expansion is also one of al-Khayyam's discoveries (Omar).

Another of al-Khayyam's contributions to mathematics was his work with parallels. He looked at Euclid's theories on parallels and from his critiques, European mathematicians later developed non-Euclidean geometry (Omar). The book al-Khayyam wrote on this was called Sharh ma ashkala min musadarat kitab Uqlidis or Explanations of the Difficulties in the Postulates of Euclid (Omar).

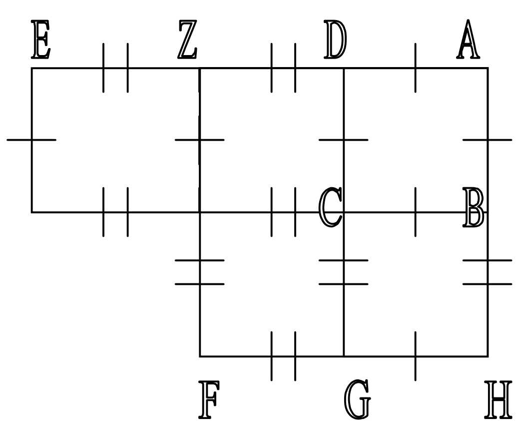
Like many great mathematicians, some of al-Khayyam's work was not original and came from studying his predecessors like al-Khwarizmi, Euclid, Appolonius, Almahani, and Abu Ja'far al-Khazin (Kasir 22). This is observed in the way al-Khayyam used Arabic letters in his geometric figures. In corresponding figures used by historical Greek mathematicians, the Arabic letters match up exactly with corresponding Greek letters (Kasir 23).

Throughout history to the time of al-Khwarizmi and around 300 years later to the time of al-Khayyam, mathematics was still written out in words. An example of the way an equation would be written in al-Khayyam's book is, "A cube and squares are equal to roots and numbers." This equation written as we know it would be "x^3 + ax^2 = bx + c" (Kasir 23). Al-Khayyam used a different way of classifying equations than we use today. Instead of classification by the highest degree, he first classified by the number of terms in the equation and then by how many terms were on each side of the equal sign. One group is called "species" and they are "simple equations" or equations with two terms such as "a = x". Another type is "compound equations" divided into "trinomial and tetranomial equations" of which trinomial might look like "x^2 + bx = a" and tetranomial "x^3 + cx^2 + bx = a" (Kasir 25). In total, he had twenty-five variations that represented all known forms of an equation up to degree three (Kasir 24-25).

Al-Khayyam's book Demonstration of Algebraic Problems has a few examples of his proofs of quadratic equations using geometry. In chapter IV on trinomial equations, he shows that the "first species" equation which is "A square and roots equal a number" or $x^2 + ax = b$ can be solved using geometry (Kasir 59-60). This is al-Khayyam's process for solving an equation of this type. "Multiply half (the coefficient) of the root by itself and add the product to the number. Then from the square root of the sum so obtained subtract half (the coefficient) of the root. The remainder is the root of the square" (Kasir 59). There are two conditions that have to be met for this process to be possible. The first is that "the number of roots must be even in order that when it is divided by two the result shall be a whole number" and that "the square of half the number of roots added to the number must be equal to a square number" (Kasir 59).

An example of this process can be done for the equation $x^2 + 10x = 39$. By following the process above, divide 10 by 2 to get 5. Then square 5 to get 25 and add 25 to the constant 39 to get 64. Then the square root of 64 is taken to be 8. Subtract the earlier halved number 5 from 8 to get 3, which is the solution.

This algebraic process was created from observing the geometric proof. "Let the square AC plus ten times the root equal thirty-nine in number. Let ten times its root represent the surface of the rectangle CE" (Kasir 59).



So (the square AC) + (the rectangle CE) = 39. DE is 10 units in length. Bisect line DE at Z so that EZ = ZD = 5. By constructing the square on ZA, you can see (EA)(AB) + (the square CF) = (ZA)^2. So then 39 + 25 = (ZA)^2 means that ZA = 8 and AD therefore equals 3.

Al-Khayyam did work in astronomy as well. In 1073 he helped build an observatory for the sultan Malik-Shan in Isfahan. He measured the length of a year to be 365.24219858156 days, which was only off by one day every five thousand years. He also proved that instead of the universe moving around earth, that the earth rotated on an axis. He is said to have proven this by making a circular room with a spinning platform with candles on the walls to represent the stars. His theory was that earth moved while the stars stayed still (Omar). He also made tables for astronomy called Ziji Malikshahi (Liukkonen5).

5. **Alhazen**

Between the time of al-Khwarizmi and al-Khayyam there was a man named Abu Ali al-Hazen ibn al-Hasan ibn al-Haytham also called Alhazen. He was born in Basra, Iraq in 965 A.D. He contributed to multiple fields including mathematics, optics, anatomy, astronomy, and engineering. He wrote over 200 works and 96 of them are scientific. More than 70 of his works have survived and half are on math, 23 are on astronomy, and 14 are on optics (Alhazen).

As an engineer Alhazen did not do so well. Alhazen believed that he could control and change how the Nile would flood. Alhazen was asked by the ruler Al-Hakim bi-Amr Allah to actually try and change the Nile. After realizing he could not do it, Alhazen pretended to be crazy so they would not kill him. From 1011 till he died in 1021, they confined him in his home where he continued his work in other sciences. At this time he wrote his greatest works. During his confinement, he wrote Book of Optics, for which he is most famously known. He proved through experimentation with tools like lenses and mirrors that light moved in straight lines. His work with optics led him into work with anatomy and the structure of eyes. He was also the first person to use a camera obscura where "light from a scene passes through the hole and strikes a surface where it is reproduced, in color, and upside-down" to "project an entire image from outdoors onto a screen indoors" (Camera). In Islam, Alhazen's the "founder of experimental psychology" (Alhazen). His work in optics led him to believe vision is actually a function of our brain and not our eyes, which led to work in perception (Alhazen).

Also in Alhazen's book, he worked on a problem that Ptolemy first asked, which is now called Alhazen's problem. As a problem in optics, the problem states, "Given a light source and a spherical mirror, find the point on the mirror where the light will be reflected to the eye of an observer." Working on this, he was able to make formulas for the "sum of fourth powers" which led to a formula for the "sum of any integral powers." This allowed him to use integration to find volume for a paraboloid. He almost had the formula to integrate any polynomial, but was only able to find "integrals for polynomials up to the fourth degree." His work was important in developing integral calculus because he found formulas for integrals up to degree four. He did solve the above problem, but with conic sections and geometry and not algebraically (Alhazen).

In mathematics, Alhazen did work with geometry, conic sections, and number theory. In geometry, he came up with the formula for adding the first 100 numbers, which he proved geometrically. He also was a developer of algebraic or analytical geometry that uses algebra and geometry together. His work with Euclid's parallel postulate influenced the work of al-Khayyam and the beginnings of non-Euclidean geometry. In number theory, Alhazen worked on perfect numbers in his book Analysis and Synthesis. He came up with the formula for perfect numbers "[2^(n - 1)][(2^n) – 1] where (2^n) – 1 is prime" but could not prove it true (Alhazen). It was later proven true by Euler in the eighteenth century (Alhazen). Alhazen also wrote a book called Opuscula on "congruences" where he uses the Chinese Remainder Theorem and Wilson's Theorem (Alhazen).

A strange area Alhazen experimented in was with psychology and musicology and its effects on animal souls. His book on this is called Treatise on the Influence of Melodies on the Soul of Animals. He experimented on horses and birds, and his book tells of an experiment with a camel to show music will cause a camel to change its pace. Experiments in this area actually continued until the 19th century (Alhazen).

6. Conclusion

Each of these Arabic mathematicians has made contributions to multiple fields of study. They all have contributed directly and indirectly through science to the advancement of mathematics. It was said for a long time that the Arabs made no significant contribution to mathematics other than to copy the work of all the great minds before them and preserve it until it could be passed on to Europe. It is now known that the Arab scientist and mathematicians did much more than translate and preserve. For hundreds of years, men like al-Khwarizmi, al-Khayyam, and Alhazen did like all the great thinkers before and after them and tried to emulate, study, and improve upon older methods while contributing many ideas of their own. During this time, once thought of a the "Dark Ages" of mathematics, it was because of men like al-Rashid and al-Mamun where their visions lead to the gathering, translating, and preservation of knowledge at Baghdad that has allowed us to know so much about Greek and Indian work from long ago (Bunt 227 and Morgan XV). We now know of the accomplishments of the Arab world and their contributions to furthering human knowledge that started around 762 A.D. with the founding of Baghdad and lasted for hundreds of years until mathematical advances could take hold again in Europe and other places.

Works Cited

"Alhazen". Wikipedia. March 16, 2009 <http://en.wikipedia.org/wiki/Alhazen>.

Boyer, Carl B. A History of Mathematics. New York: John Wiley & Sons, Inc., 1991.

Bunt, Lucas N.H., Philip S. Jones, and Jack D. Bedient. The Historical Roots of Elementary Mathematics. New York City: Dover Publications, Inc., 1988.

"Camera". Wikipedia. April 27, 2009 < http://en.wikipedia.org/wiki/Camera_obscura>.

Fitzgerald, Edward. Rubaiyat of Omar Khayyam. New York City: Barse & Hopkins Publishers, 1917.

Kasir, Daoud S. Columbia University Contributions to Education Teachers college Series. 385 vols. New York City: Bureau of Publications, 1931.

Liukkonen, Petri. "Omar Khayyam (1048-1131) ". March 16, 2009 <http://www.kirjasto.sci.fi/khayyam.htm >.

Morgan, Michael. Lost History: The Enduring Legacy of Muslim Scientists, Thinkers, and Artists. Washington D.C.: National Geographic, 2007.

"Omar Khayyam". New World Encyclopedia. March 16, 2009 <http://www.newworldencyclopedia.org/Omar_Khayy%C3%A1m>.

Smoller, Laura. "Did you know...?". UALR Department of History. March 16, 2009 <http://ualr.edu/lasmoller/aljabr.html>.