Chinese Mathematics

Thanks to Mark Lane

Magic Squares

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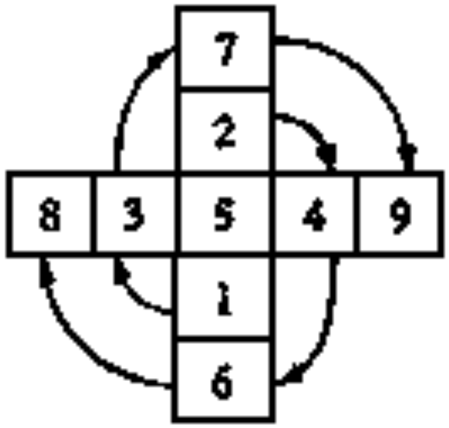
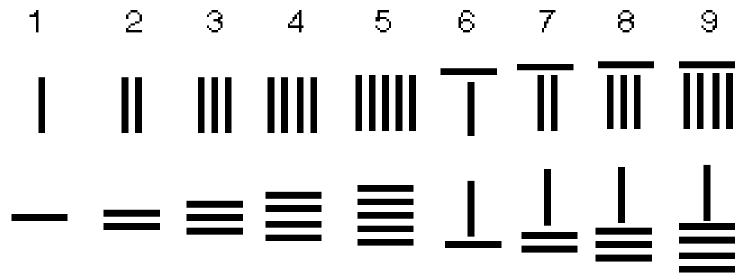
There are two types:

- Lo Shu (tortoise)
- Ho Thu (dragon horse)



Rod Numerals



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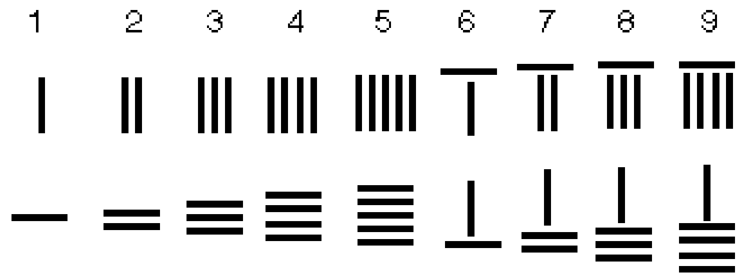

These were used 1000 - 500 BCE.
0 was added in the 13th century - Ch'in (1274) was the first Chinese to give a separate symbol, a circle, for zero.

Chiu-Chang Suan-Shu
Arithmetic in Nine Sections

- 1000 BCE - 200 CE
- Textbook consisting of 246 problems
  - Problems and rules for obtaining solutions
  - Omission of solution explanations
- Nine Chapters
  - Surveying and Engineering Formulas
  - Taxation and Bureaucratic Administration
  - Specific Computational Techniques

1. Fang Thien
(Land Surveying)

- Algorithms for finding areas of figures.
  - Use 3 as an approximation for π

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$$\frac{98}{182} \to \boxed{\frac{49}{91}} \to \frac{49}{42} \to \frac{7}{42} \to \frac{7}{35} \to \frac{7}{28} \to \frac{7}{21} \to \frac{7}{14} \to \frac{7}{7} = \frac{7}{13}$$

4. Shao kuang
(Short width)

- Finding sides of figures
  - Use of square and cube roots
    - Approximations found but decimals uncertain

5. Shang kung
(Civil engineering)

- Formulas for computing volumes for construction purposes

Taxation and Bureaucratic Administration

2. Su Mi
(Millet and Rice)

- Simple percentage and proportions
- Used for grain exchange

3. Tshui Fen
(Distribution by Proportion)

- Methods of proportion
- “Rule of Three”
  - Indian origin - Sanskrit version 628 CE

$$\frac{35}{6} = \frac{x}{9}$$

6. Chun shu
(Impartial Taxation)

- Pursuit problems
  - Used to calculate travel costs

7. Ying pu tsu
(Excess and Deficit)

Rule of false position.
We want to solve $ax - b = y$.
Example: $6x - 12 = y$.
Make two guesses a and b.

$$a = 3(\text{excess}) \text{ and } b = 1(\text{deficit})$$

$$6(3) - 12 = 6 = w$$

$$6(1) - 12 = -6 = v$$

Then

$$x = \frac{wb - va}{w - v} = \frac{6 \cdot 1 - (-6) \cdot 3}{6 - (-6)} = \frac{24}{12} = 2$$

8. Fang ch'eng
(Calculation by Square Tables)

- Solving systems of equations
- Used counting rods
  - Black - positive
  - Red - negative
- Elementary row and column operations
  - Gauss elimination

Chapter 8 Example

There are three grades of corn. Neither two baskets of the first grade, three baskets of the second, nor four baskets of the third grade taken separately comprise a full required measure. If, however, one basket of the second grade were added to the first grade corn, one basket of the third grade corn added to the second, and one basket of the first grade to the third grade baskets, then the grain would comprise one required measure in each case. How many measures of the various grades does each mixed basket contain?

$$2x + y = 1$$

$$3y + z = 1$$

$$x + 4z = 1$$

1st grade (x)	1		2
2nd grade (y)		3	1
3rd grade (z)	4	1	
Required measure	1	1	1

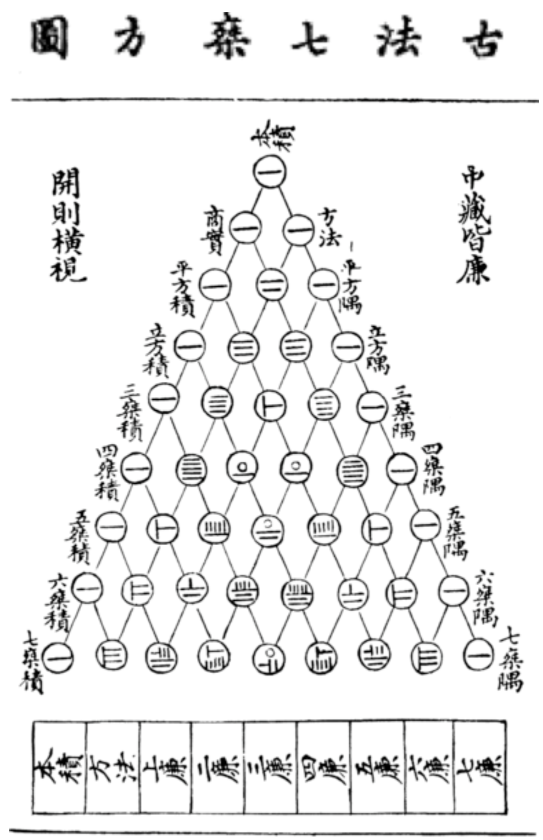
gives the matrix:

2	1	0	1
0	3	1	1
1	0	4	1

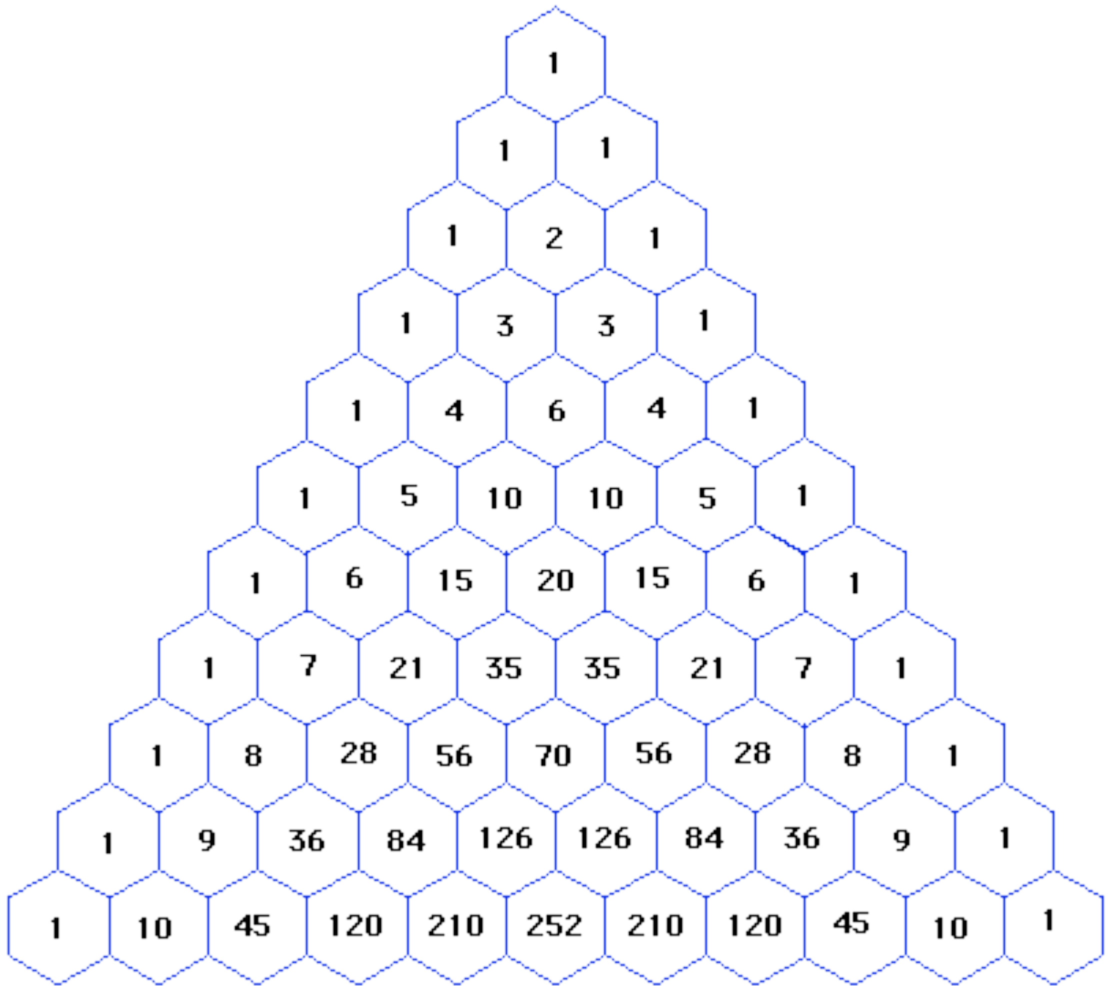
9. Kou Ku
(Right Triangles)

- Understanding and use of the Pythagorean Theorem
  - Hsien - hypotenuse

Yang Hui (books in 1261 and 1275) gave the earliest extant presentation of Pascal's triangle.



Pascal's Triangle



The Pascal triangle (actually known long before Pascal) is a table of the binomial coefficients where the (n, k)th entry is

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Named after Blaise Pascal (1623 - 1662), a French mathematician.

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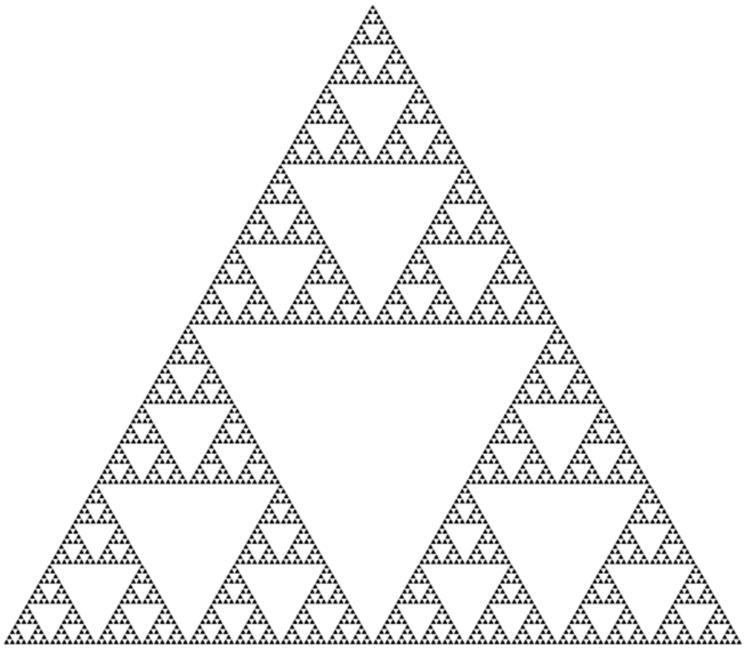
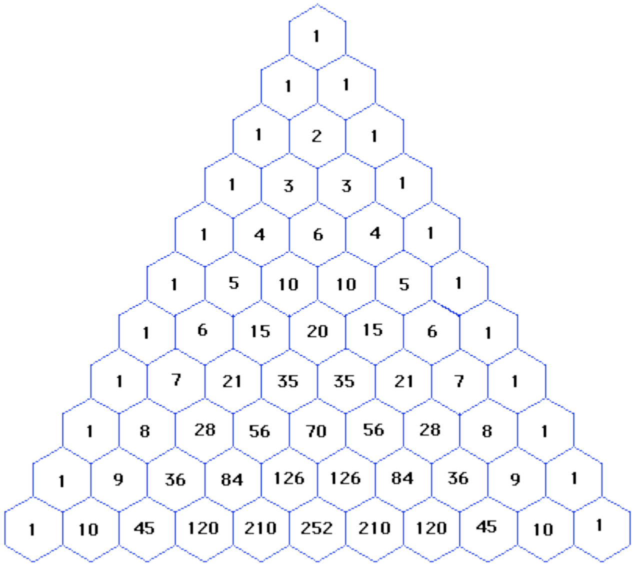
- Moving inwards, the next pair of diagonals contain the triangular numbers in order.

Coloring the odd and even numbers with different colors to distinguish them results in a pattern known as Sierpinski triangle, which is a fractal. Shading all multiples of 3, 4, etc., results in other patterns and combinations.



There are also more surprising, subtle patterns. From a single element of the triangle, a more shallow diagonal line can be formed by continually moving one element to the right, then one element to the bottom-right, or by going in the opposite direction. An example is the line with elements 1, 6, 5, 1, which starts from the row 1, 3, 3, 1 and ends three rows down. Such a "diagonal" has a sum that is a Fibonacci number.



Modular Arithmetic

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So what number between 0 and 6 is 24 equivalent to mod 7?

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So

$$24 = 3 \mod 7$$

Notice that $24 - 3 = 21$ which is divisible by 7.

Chinese Remainder Theorem

http://www.cut-the-knot.org/blue/chinese.shtml

According to D.Wells, the following problem was posed by Sun Tsu Suan-Ching (4th century AD):

There are certain things whose number is unknown. Repeatedly divided by 3, the remainder is 2; by 5 the remainder is 3; and by 7 the remainder is 2. What will be the number?

Oystein Ore mentions another puzzle with a dramatic element from Brahma-Sphuta-Siddhanta (Brahma's Correct System) by Brahmagupta (born 598 AD):

An old woman goes to market and a horse steps on her basket and crashes the eggs. The rider offers to pay for the damages and asks her how many eggs she had brought. She does not remember the exact number, but when she had taken them out two at a time, there was one egg left. The same happened when she picked them out three, four, five, and six at a time, but when she took them seven at a time they came out even. What is the smallest number of eggs she could have had?

A simple form of the Chinese Remainder Theorem

If m and n are relatively prime positive integers (i.e. gcd $(m, n) = 1$) and $a, b \in \mathbb{Z}$ then the system

$$x \equiv a \mod m$$

$$x \equiv b \mod n$$

has a solution.

The proof gives the algorithm by finding u and v so that $mu + nv = 1$ (this can be done by the Euclidean Algorithm). Then $x = a + mu(b - a)$ is a solution of the system.

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Solve the system

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Since $gcd(4, 7) = 1$ we can write 1 as a integer combination of 4 and 7. It is easy to see that

$$4(2) + 7(-1) = 1,$$

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Check this!

Mathematics In India

Hindu Mathematics

Bhaskara II (1114 - 1185)

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In many ways Bhaskaracharya represents the peak of mathematical knowledge in the 12th century. He reached an understanding of the number systems and solving equations which was not to be achieved in Europe for several centuries.

Lilavati was the name of Bhaskaracharya's daughter. From casting her horoscope, he discovered that the auspicious time for her wedding would be a particular hour on a certain day. He placed a cup with a small hole at the bottom of the vessel filled with water, arranged so that the cup would sink at the beginning of the propitious hour. When everything was ready and the cup was placed in the vessel, Lilavati suddenly out of curiosity bent over the vessel and a pearl from her dress fell into the cup and blocked the hole in it. The lucky hour passed without the cup sinking. Bhaskaracharya believed that the way to console his dejected daughter, who now would never get married, was to write her a manual of mathematics!

http://www-groups.dcs.st-and.ac.uk/ history/Biographies/Bhaskara__II.html

This is a charming story but it is hard to see that there is any evidence for it being true. It is not even certain that Lilavati was Bhaskaracharya's daughter. There is also a theory that Lilavati was Bhaskaracharya's wife.

The topics covered in the thirteen chapters of the book are: definitions; arithmetical terms; interest; arithmetical and geometrical progressions; plane geometry; solid geometry; the shadow of the gnomon; the kuttaka; combinations.

One of Bhaskaracharya's methods for multiplying proceeds as follows:

325

243

Multiply the bottom number by the top number starting with the left-most digit and proceeding towards the right. Displace each row one place to start one place further right than the previous line. First step

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325

243

729

Second step

Second step

325

243

729

486

Third step, then add

Third step, then add

325

243

729

486

1215

78975

In his conclusion to *Lilavati* Bhaskaracharya writes:
Joy and happiness is indeed ever increasing in this world for those who have Lilavati clasped to their throats, decorated as the members are with neat reduction of fractions, multiplication and involution, pure and perfect as are the solutions, and tasteful as is the speech which is exemplified.

Comparisons with others

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“. . .Hindu mathematics remained largely a handmaiden to astronomy. With the Greeks, mathematics attained an independent existence and was studied for its own sake. Also, as a result of the caste system, mathematics in India was cultivated almost entirely by the priests; in Greece, mathematics was open to any one who cared to study the subject.” [p.229]

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