Japanese History

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Abstract

This paper will discuss early Japanese mathematical development from the earliest ages to the methods of the Wasan in Japan today. Included is a brief biography of Seki Takakazu, one of the most respected Japanese mathematicians in modern history.

1 History of Japanese Mathematics

1.1 BC-552 AD : The Beginning

It is difficult to pinpoint the birth of mathematics in Japan. It would be like trying to uncover English history without all of the information given to us in the Roman texts. It would be very difficult to retrieve information from 'native sources,' because of their remote environment at the time. The Japanese had the same problem as the early English, but they did not have the Romans to write for them. The Babylonians and Egyptians had advanced writing techniques which enabled them to record most of their scientific and technological breakthroughs. Unfortunately, there is only one true piece of evidence documenting the very early mathematics of the Japanese, and this is one of their number systems. We are told that in the Izanagi-no-Mikoto reign, the Mikados composed a system of numeration that extended numbers to very high powers of ten. The interesting part is that this system embodied the same exponential laws used by Archimedes in his *Sand Reckoner.* The equation is as follows:

$$a^m * a^n = a^{m+n}$$

For Example:

$$2^2 * 2^3 = 2^{2+3}$$

Figure 1: Picture of Ancient Calendars, [1].

$$4 * 8 = 2^5$$
$$32 = 32$$

The table in Figure 1 is a Japanese anthropologist's representation of what it might have looked like, but is only given as a possibility, [1].

These advanced systems were not typically used by trade people (because the huge numbers were unnecessary)but to the scholars they were significant, for this was the beginning of modern number names. There is evidence of the base yorozu, or Greek base myriad in India, and we know that it still persists today in China. Take, for instance, our word for 10^5. It is not a hundred thousand, but actually ten myriads. Similarly, our million is actually a hundred myriads. It is to be expected that, with more investigation, we will uncover more information about their observation of the stars and the way of mensuration in these early periods, but for now, we can only hypothesize about these ideas, [1].

1.2 552-1600 : The Dark Ages

Buddhism is accredited with the enlightenment of Japan in this time period. It is in 522 when a man from China, Szŭ-ma Ta (the Japanese called him Shiba Tatsu), set up a shrine of Buddha and began to preach his religion. Two years later, when Buddhism took a stronghold in Japan, Wang Pao-san, a master of the calendar, and Wang Pao-ling, a master astrologer and master of chronology, made known the Chinese chronological system. With this knowledge, the crowned prince Tenchi (Tenji) invented a water clock and later split the day into 100 hours. Upon gaining control of the kingdom he founded a school in which two doctors and twenty students resided. Along with this school, an observatory was also established, and for the first time mathematics was recognized in Japan, [1].

The official records show that a university system was established by Monbu in 701 A.D. These works were regulated by the higher institutions of learning and the nine works are as follows:

1. *Chou-pei Suan-ching*: One of the oldest books on mathematics, it is labeled the so-called "Pythagorean Theorem" of today. From the word "Chow Pei" (equivalent to "Chou-pei") we get "Thigh bone of Chow,"which probably signifies, from its shape, altitude and base- a right triangle. Interestingly, this book was written 500 hundred years before Pythagoras studied it.

2. *Sun-tsu Suan-ching*: This composition actually consisted of three books and is commonly known as the "Swan-king" or "classic Arithmetic". Sun-tsu describes algebraic equations like the following:

   The problem is to "find a number which, when divided by 3, leaves a remainder of 2, when divided by 5, leaves a 3, and when divided by 7, leaves a 2,"[1].

   This question asks us to play with remainders of numbers. A simplified version of the above question is as follows:

   $$3 \mod 5 = ?$$

   $$5 \mod 3 = ?$$

   $$7 \mod 2 = ?$$

   The number 23 is the number we are looking for because 23 divided by 3 equals 21 with a remainder of 2, 23 divided by 5 equals 20 with a remainder of 3, and 23 divided by 7 equals 21 with a remainder of 2.

3. *Liu-Chang*: Information about this work is unknown.

4. *San-k'ai Chung-ch'a*: Information about this piece is also unknown.

5. *Wu-t'sao Suan-shu*: The author and precise date of this work is unknown but seems to have been written between 200 and 300 A.D., and is one of the standard treatises on arithmetic.

6. *Hai-tao Swan-shu*: This work seems to be a reproduction of the *San-k'ai Chung-ch'a* (number four in this list) and is related to the arithmetic needed to find the distance between islands in Japan.

7. *Chiu-shu*: This work is also lost to us.

8. *Chiu-chang Suan-shu*: "Arithmetical Rules in Nine Sections" is regarded today as the the best Arithmetic classic of China. This classical work had an important effect on the Japanese, and is composed of over 246 problems arranged into nine sections, respectively listed below.

    (a) *Fang-tien*: Relates to the mensuration of many plane figures. This collection includes triangles, circles, and quadrilaterals. The work also hinted at fractions.

    (b) *Suh-pu*: This mostly describes the "Rule of Three" which is an ancient method of solving proportions; for instance, when three numbers (a, b, and c) are given, and you want to find d such that a:b::c:d. [**?**]

    (c) *Shwai-fên*: This work mostly discusses partnership.

    (d) *Shao-kang*: Relates to the extraction of roots and squares; This process is very similar to techniques used today.

    (e) *Shang-kung*: Describes the measurement of prisms, cylinders, pyramids, and other circular objects such as cones.

    (f) *Kin-shu*: Describes alligation, or rule of mixture, which is an arithmetic way of solving mixture problems; Following is an example of a medial alligation problem.

> "Suppose you make a cocktail drink combination out of 1/2 Coke, 1/4 Sprite, and 1/4 orange soda. The Coke has 120 grams of sugar per liter, the Sprite has 100 grams of sugar per liter, and the orange soda has 150 grams of

sugar per liter. How much sugar does the drink have? This is an example of alligation medial because you want to find the amount of sugar in the mixture given the amounts of sugar in its ingredients. The solution is just to find the weighted average by composition,"[5].

$$\frac{1}{2} \cdot 120 + \frac{1}{4} \cdot 100 + \frac{1}{4} \cdot 150$$

The total amount of sugar will be 122.5 grams, because $60 + 25 + 37.5 = 122.5$g.

(g) *Ying-pu-tsu*: This book discusses the rule of false position, or an ancient attempt at using guess and check to solve equations.

(h) *Fang-ch'êng*: This chapter discusses linear equations. For example:

"If 5 oxen and 2 sheep cost 10 taels of gold, and 2 oxen and 8 sheep cost 8 taels of gold what is the price of each?"[1]

$$5o + 2s = 10$$

$$2o + 8s = 8$$

Using substitution for $s = \frac{10-5o}{2}$ into the bottom equation, you get $40 - 20o + 2o = 8$, so $18o = 32$, so o equals $\frac{32}{18}$. When you plug this back into the second equation and reduce fractions you compute that,

$$s = \frac{5}{9} \text{ and } o = \frac{16}{9}.$$

As the above problem illustrates, it is probable that this chapter discusses the use of negative numbers.

(i) *Kou-ku*: This chapter is a discussion of the Pythagorean Theorem and is defined as "The first side and the second side being each squared and added, the square root of the sum is the hypotenuse,"[1]. One of the 24 problems in this section asks you to solve...

$$x^2 + (20 + 14)x - 2 \cdot 20 \cdot 1775 = 0$$

If the text is interpreted correctly, and there is no tampering with the dates, it would make this the first known use of the quadratic formula, [1].

This chapter refers to π as being equal to 3. However, later mathematicians changed this to $\frac{157}{50}$ which is equal to 3.14.

9. *Chui-shu* This book is lost to us, and is only known by name.

This list includes the most important Chinese classics and shows an important attempt to integrate this into Japanese schools at the opening of the 8^{th} century,[1].

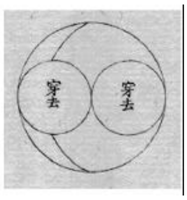
1.3 600-1875 : Era of Intellectual Awakening

When mathematics developed during this time, they called it Wasan. This knowledge was gained through the literature brought back from China. The Wasan books were obtained when Hideyoshi, ruler of Japan at this time, invaded the Korean peninsula. There are two customs which accelerated the development of the Wasan, [3] pg 79. The first is *idai*, or the challenging problems at the back of a Wasan book. Wasan mathematicians would solve them and then publish these problems, but would add some of their challenging problems in a new book. The second custom was *sangaku*, which was a wooden tablet of mathematics. People would use these tablets to present interesting properties or to solve difficult problems. Once finished, they would hang these at a shrine and would dedicate their work to the shrine they had built. The Japanese would also hang these tablets under their own roofs. Most of these problems were geometric and the pictures were in full color and very beautiful. In general, Wasan covers analysis, number theory, combinations, and some geometry. Along with the areas of circles, arc lengths, and magic squares, they studied astronomy and the art of divination. Since the Wasan books were written in the form of Chinese Mathematic books, they treated only particular examples. For example, ellipses and rhombi were studied but hyperbolas/parabolas and parallelograms were not discussed.

In the early days of the Wasan, the area of circles and arc length were very popular problems. One such problem is given to us by Viviani (reference Figure 2,

"Given that a sphere of radius r is bored by two cylinders with radii $\frac{r}{2}$ as in the figure. Find the volume and surface area of the remaining part,"[3].

Figure 2: Viviani's Problem with Spheres [3]



Today, most of the Wasan books are written in Chinese, making it very difficult for the Japanese to understand. These books are extremely scarce, causing them to be very expensive. This is a sad circumstance, because these books are very valuable and yet they are unable to reach Japanese classrooms today, [3].

2 Japanese Credited Mathematician

2.1 Seki Takakazu

Seki (as seen in Figure 3)was born into a samurai warrior family in March of 1642. He was adopted into a noble family by the name Seki Gorozayemon. This is where Seki derived his name from which he chose over the name of his birth parents. Seki was introduced to this idea of mathematics from a talented servant inside his household. He was self educated in mathematics, which made him an infant prodigy of the time. Growing up, Seki acquired a library of both Japanese and Chinese books on mathematics, and soon was acknowledged as an expert. Much secrecy surrounded the schools in Japan, as it is difficult to find the contributions of his work as a teacher. It is known that he was credited for major contributions in Calculus which he later passed on to his pupils. For example, in 1674, he published *Hatsubi Sampo,* in which he solved a series of fifteen problems which were posed 4 years prior. His unique and careful analysis of the problems was one of the many reasons for his success as a teacher. Seki was the first person to study determinants in 1683. A decade later, Leibniz independently used these determinants to solve simultaneous equations, but Seki's version was more general. He examined equations on both negative and positive roots, yet he did not understand the concept of complex numbers. He wrote on magic

Figure 3: Seki Takakazu [2]

squares, and was the first Japanese mathematician to do so. In 1685 he solved the equation $30 + 14x - 5x^2 - x^3 = 0$ using the same method used by Horner a century later. Among his greater works were the Diophantine equations. For example, he considered possible integer solutions to $ax - by = 1$ where a and b are integers,[2]. Seki was known as "The Arithmetical Sage," which was later engraved on his tombstone when he died in October of 1708. His take on life can best be described as follows:

> "In due time he, as a descendant of the samurai class, served in public capacity, his office being that of examiner of accounts to the Lord of Koshu, just as Newton became master of the mint under Queen Anne. When his lord became heir to the Shogun, Seki became Shogunate samurai and in 1704 was given a position of honor as master of ceremonies in the Shogun's household,"[2].

3 Conclusion

To reiterate, I have briefly discussed the transition of Japanese Mathematics from its ancient to its modern ideas and practices. I described the early development of word and number and their early uses with the first settlers of Japan. I went on to describe the establishment of Buddhism in Japan and how it brought forth the first school in the dark ages. I also discussed the importance of the books that were published from this school. I continued my review with the Era of Intellectual Awakening. Most influential in this era was the Wasan including the discussions of the *idai*, and the *sangaku.* I finished my tour of Japanese Mathematics with a brief look into the life of Seki Takakazu. His many contributions to the field of Japanese Mathematics are still used today. Studying the advancements of Japanese mathematicians through their distinguished history has given me a greater appreciation of the field I am studying, one that I rarely associate with pre-modern cultures or their peoples.

References

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