Math 467
Spring 2007 Name:
Exam 1 – Ch. 1 - 5

True / False. Circle T if the statement is always true and F if the statement is sometimes false.

1. T F There is evidence that the Babylonians understood how to calculate exponentials as long ago as 300 AD.
2. T F Pythagoras is the first mathematician discussed by his name in the history of mathematics.
3. T F Pythagoras formed the first mathematics school around 518 BC.
4. T F Both men and women were allowed to be members of Pythagorean society.
5. T F It is not possible to trisect an angle.
6. T F π is an algebraic number.
7. T F Ludolph van Ceulen was the first to use trigonometric methods to improve the perimeter techniques of estimating π.
8. T F Euclid's *Elements* was meant to be a complete reference for all known plane and solid geometry at the time.
9. T F According to "The Great π/e Debate", e was discovered when the Venutians came to Earth.

Multiple Choice. Circle the letter corresponding to the best answer for each of the following questions.

10. In "The Great π/e Debate," the person arguing for π was

    A. Tom Garrity.
    B. Colin Adams.
    C. Ed Burger.
    D. Paul Erdös.

11. Which of the following is **not** a root of the word zero?

    A. the Latin word *zephirum*.
    B. the Arabic word *sifr*.
    C. the German root *cifra*.
    D. none of the above.

12. Where is the first mention of irrational numbers?

    A. with the Pythagorean Hippasus of Metapontum in ancient Greece
    B. with the study of irrational ratios developed by Plato's friend Eudoxus
    C. in the Indian *Sulba Sutras* composed between 800-500 BC
    D. with de Moivre in the eigtheenth century

13. Which of the following describes the Hindu-Arabic numeral system?

    A. simple grouping system
    B. multiplicative grouping system
    C. ciphered numeral system
    D. positional numeral system

14. Which of the is **not** one of the reasons the agricultural revolution allowed more scientific and mathematical advances than the stone age?

    A. Need for new technologies
    B. Development of war
    C. Sedentary lifestyle
    D. Creation of leisure time
    E. New forms of political organization

15. Which of the following is **not** one of the beliefs of the Pythagorean society?

    A. At its deepest level, reality is mathematical in nature.
    B. The souls of the dead might appear on Earth again, not only as a new human, but also animals.
    C. The study of numbers and numerical relationships, and of sounds and harmonic relationships leads to personal perfection.
    D. The soul is finite, and is only allowed a certain number of lifetimes.
    E. Certain symbols have a mystical significance.

16. Which of the following is **not** one of Euclid's postulates?

    A. Any two points can be joined by a straight line.
    B. Any curve can be extended indefinitely.
    C. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
    D. All right angles are congruent.
    E. If two lines intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough.

17. The number 16 is

    A. perfect.
    B. deficient.
    C. abundant.
    D. amicable.
    E. transcendental.

Short Answer. Answer each of the following questions. Show all work and explain your reasoning to receive full credit.

18. (a) Convert $(345)_7$ into base 10.
    (b) Convert $(457)_{10}$ into base 8.
    (c) Calculate $(1234)_7 + (345)_7$.

19. (a) What is a unit fraction?
    (b) Are fraction decompositions into unit fractions unique? Explain your answer and give an example to justify your conclusion.

20. (a) Show that for any positive integer n, $\frac{2}{n}$ can be expressed by the sum $\frac{1}{n} + \frac{1}{2n} + \frac{1}{3n} + \frac{1}{6n}$.
    (b) Use this to find a decomposition of $\frac{2}{101}$ into unit fractions.

21. (a) State the Pythagorean Theorem, including all hypotheses.
    (b) Give a proof of the Pythagorean Theorem.
    (c) (Bonus) To whom is the proof you gave above attributed?

22. (a) Find the $\gcd(10835, 5609)$.
    (b) What is the name of the algorithm you used to answer part (a)?
    (c) Find p and q so that $\gcd(10835, 5609) = 10835p + 5609q$

23. (a) Describe the square numbers.
    (b) Give a formula for the n-th square number.
    (c) Describe the pentagonal numbers.
    (d) Give a formula for the n-th pentagonal number.

24. Prove that $\sqrt{3}$ is irrational.

25. Prove that there are an infinite number of prime numbers.