Math 467
Spring 2009
Homework for Chapters 8-9
(and some extra problems from Chapter 7)

1. What are Euclid's five axioms? What are Euclid's five postulates? Which of the five postulates is eliminated in non-Euclidean geometry?

2. (∗) A man wishes to go from his house to the bank of a straight river for a pail of water, which he will then carry to his barn, on the same side of the river as his house. Find the point on the riverbank that will minimize the distance the man must travel.

3. After staining the holy chaplet of fair-eyed Justice that I might see thee, all-subduing gold, grow so much, I have nothing, for I gave forty talents under evil auspices to my friends in vain, while, O ye varied mischances of men, I see my enemies in possession of the half, the third, and the eighth of my fortune. How many talents did the unfortunate man once possess?

4. (∗) The 3 Graces were carrying baskets of apples, and in each was the same number. The 9 Muses met them and asked each for applies and they gave the same number to each Muse and the 9 and the 3 each had the same number. Tell me how many they gave and how they all had the same number.

5. (∗) Solve the following problem from Diophantus: Find 4 numbers, the sum of every arrangement 3 at a time being given; say 22, 24, 27, and 20.

6. (a) Establish the identities

$$(a^2 + b^2)(c^2 + d^2) = (ac \pm bd)^2 + (ad \mp bc)^2.$$

   (b) Use these identities to express 481=(13)(37) as the sum of 2 squares in 2 different ways.

7. Use the Sieve of Erastothenes to find all prime numbers less than 100.

8. (∗) Use a geometric proof to show that "If a straight line is cut at random, then the sum of the rectangles contained by the whole and each of the segments equals the square on the whole." What algebraic fact have you just proved?

9. (∗) Prove: If a straight line passing through the center of a circle bisects a straight line not passing through the center, then it also cuts it at right angles; and if it cuts it at right angles, then it also bisects it.