

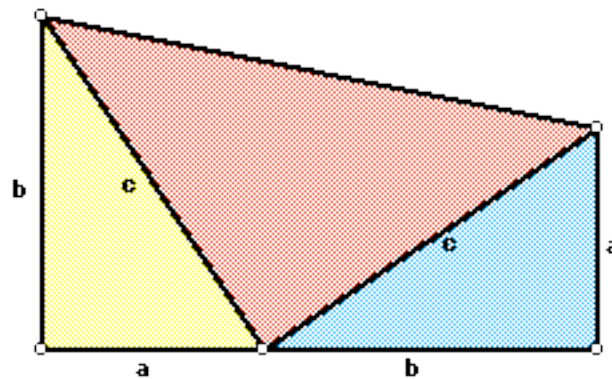
Math 467
Spring 2009
Homework for Chapter 7

Definition 1 Let a and b be integers. The greatest common divisor of a and b is denoted by $\gcd(a, b)$ and is the natural number n that satisfies the following two conditions:

1. n divides both a and b
2. If d is an integer that divides both a and b then d divides n .

Definition 2 The Euclidean Algorithm - Divide the larger of the two positive integers by the smaller one, then divide the divisor by the remainder. Continue this process, of dividing the last divisor by the last remainder, until the division is exact. The final divisor is the sought gcd of the two original positive integers.

1. Use the Euclidean algorithm to find the gcd of 5913 and 7592.
2. Use the Euclidean algorithm to find the gcd of 1827, 2523, and 3248.
3. (*) Find integers m and n so that $\gcd(5913, 7592) = 5913m + 7592n$.
4. (*) Use the following figure to prove the Pythagorean Theorem. This proof was first published by President Garfield in the *New England Journal of Education*.



5. Assume that alternate interior angles formed by a transversal cutting a pair of parallel lines are equal. Use this to prove:
 - (a) The sum of the angles of a triangle is equal to a straight angle.
 - (b) (*) The sum of the interior angles of a convex polygon of n sides is equal to $n - 2$ straight angles.
6. (*) Assume that the sum of the angles of a triangle is equal to a straight angle. Prove that an exterior angle of a triangle is equal to the sum of the 2 remote interior angles.
7. (*) Assume that the area of a rectangle is given by the product of its dimensions. Prove that the area of a parallelogram is equal to the product of its base and altitude.
8. Continue this line of proof. Prove that the area of a triangle is equal to half the product of any side and the altitude on that side.