Math 467
Spring 2009
Homework for Chapters 5-6

1. Hippocrates of Chios (ca. 440 BC) squared certain lunes, perhaps hoping that his investigations might throw some light on the quadrature problem. Let AOB be a quadrant of a circle. On AB as a diameter, draw a semicircle lying outside the quadrant. Show that the lune bounded by the quadrant and the semicircle has the same area as triangle AOB.

2. $(*)$ Using only a compass and an unmarked straightedge, construct an equilateral triangle upon a finite given straight line.

3. Using only a compass and an unmarked straightedge, from a given point, draw a straight line equal to a given straight line.

4. Let AOB be any central angle in a given circle. Through B, draw a line BCD, cutting the circle again at C, and AO produced in D, such that $CD = OA$, the radius of the circle. Then show that the angle $ADB = \frac{1}{3}$ angle AOB. This is a solution of the trisection problem.

5. $(*)$ Archimedes derived the formula

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

   Prove this formula.

6. A Pythagorean triple is a set of three numbers (x, y, z) that make $x^2 + y^2 = z^2$ true. One proposed way of finding these triples is to let $x = 2n + 1$, $y = 2n^2 + 2n$ and $z = 2n^2 + 2n + 1$.

   (a) Prove that this does indeed give a Pythagorean triple.

   (b) $(*)$ Find all such triples with hypotenuse less than 100.

7. $(*)$ Find all right triangles with sides of integral length whose areas are equal to their perimeters.

8. $(*)$ Verify that $(3, 4, 5)$ is the only Pythagorean triple involving consecutive integers.