

Math 467  
Spring 2009  
Homework for Chapter 4

**Definition 1** A number is perfect if it is equal to the sum of its proper divisors.

**Definition 2** A number is deficient if it exceeds the sum of its proper divisors.

**Definition 3** A number is abundant if it is less than the sum of its proper divisors.

1. Classify the numbers from 2 - 20 as perfect, deficient, or abundant.
2. Euclid proved that if  $2^n - 1$  is a prime number then  $2^{n-1}(2^n - 1)$  is a perfect number. All of these perfect numbers are even. Find the fourth perfect number predicted by Euclid's formula.
3. (\*) Prove that the sum of the reciprocals of **all** the divisors of a perfect number is 2.

**Definition 4** A pair of numbers is amicable or friendly if each number is the sum of the proper divisors of the other.

4. Show that 284 and 220 are amicable.
5. Show that Fermat's pair (discovered in 1636) of amicable numbers (17,296 and 18,416) are actually amicable.
6. Nicolo Paganini claimed that 1184 and 1210 are amicable. Show that they are.
7. (\*) Find the 21 abundant numbers less than 100. Notice that they are all even.
8. Not all abundant numbers are even. To prove this, show that 945 is abundant.
9. List the first four hexagonal numbers.
10. An oblong number is the number of dots in a rectangular array having one more column than rows. Show, geometrically and algebraically, that the sum of the first  $n$  positive even numbers is an oblong number.
11. Show, algebraically and geometrically, that any oblong number is twice a triangular number.
12. Show that 8 times any triangular number, plus 1, is a square number.
13. (\*) Prove that no isosceles right triangle exists whose sides are integers.
14. (\*) Show that  $\sqrt{2}$  is irrational.
15. Prove that the sum of a nonzero rational number and an irrational number is an irrational number.