

Math 467  
Spring 2009  
Homework for Chapter 21  
ALL PROBLEMS ARE TO BE HANDED IN

1. Euler conjectured that for  $n > 2$  at least  $n$   $n$ th powers are required to provide a sum which is itself an  $n$ th power. In 1966, L. J. Lander and T. R. Parkin, using high speed computers, discovered that

$$27^5 + 84^5 + 110^5 + 133^5 = 144^5.$$

Check the truth of this counterexample.

2. Explain the following paradox that bothered mathematicians of Euler's time: Since

$$(-x)^2 = (x)^2$$

we have

$$\log(-x)^2 = \log(x)^2,$$

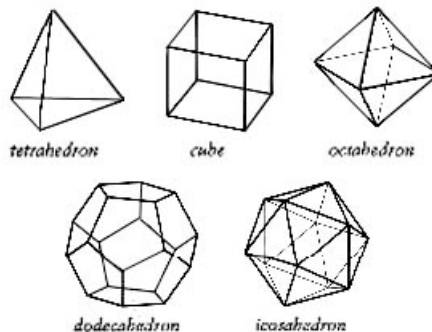
whence

$$2 \log(-x) = 2 \log x,$$

and therefore

$$\log(-x) = \log x.$$

3. A rectangular polyhedron is a geometric solid, all of whose lateral faces are identical regular polygons, and in which the same number of polygons meet at each vertex. There are only five regular polyhedrons, or Platonic solids, named according to the number of faces each has, as shown below.



In 1751, Euler proved a formula relating the faces, edges, and vertices of any convex polyhedron, not just a regular one; the result was already known to Descartes a century earlier. For the solids pictured, confirm Euler's formula

$$F + V = E + 2$$

where  $F$ ,  $E$ , and  $V$  denote the number of faces, edges, and vertices of the solid, respectively.