

Math 467  
Spring 2009  
Homework for Chapters 10-11

1. Prove the following facts from Archimedes' *On the Sphere and Cylinder*:
  - (a) The surface area of any sphere is equal to four times the area of a great circle of the sphere.
  - (b) The volume of any sphere is equal to four times the volume of the cone whose base equals a great circle of the sphere.
  - (c) (\*) If a sphere is inscribed in a right circular cylinder whose height is equal to the diameter of the sphere, then the volume of the cylinder is  $\frac{3}{2}$  the volume of the sphere.
2. (\*) Find three numbers such that the sum of any pair exceeds the third by a given amount: say the given excesses are 20, 30, and 40.
3. Find two numbers so that their sum and product are given numbers: say that their sum is 20 and their product is 96.
4. Find two square numbers having a given difference, say their difference is 60.
5. Which of the following diophantine equations cannot be solved? Explain.
  - (a)  $6x + 51y = 22$
  - (b)  $33x + 14y = 115$
  - (c)  $14x + 35y = 93$
6. (\*) Find two numbers such that the square of either added to the sum of both gives a square.
7. Find two numbers such that their difference and also the difference of their cubes are given numbers: say their difference is 6 and the difference of their cubes is 504.
8. (\*) (Alcuin of York) A hundred bushels of grain are distributed among 100 persons in such a way that each man receives 3 bushels, each woman 3 bushels, and each child half a bushel. How many men, women, and children are there?
9. (Fibonacci) Find a multiple of 7 having the remainders of 1, 2, 3, 4, and 5 when divided by 2, 3, 4, 5, and 6.
10. (\*) (Euler) Divide 100 into two summands such that one is divisible by 7 and the other by 11.