Math 467
Spring 2009
Homework for Chapters 1 - 3

Base arithmetic

In Base 10, the number 1,987,654 means

1	9	8	7	6	5	4
millions	hundred thousands	ten thousands	thousands	100s	10s	1s

In Base 5, we are only allowed to use digits 0, 1, 2, 3, 4. For example, 1, 032,404 in base 5 means:

1	0	3	2	4	0	4
15,625s	3125s	625s	125s	25s	5s	1s

So in base 10, $(1032404)_5$ is

$$4 + 0(5) + 4(25) + 2(125) + 3(625) + 0(3125) + 1(15625)$$

$$= 4 + 100 + 250 + 1875 + 15625$$

$$= 17854$$

Therefore $(1032404)_5 = (17604)_{10}$

Converting Bases

Write $12,345$ in base 5. First, notice that the powers of 5 are $1, 5, 25, 625, 3125, 15625, \ldots$, so in base 5, the number will look like

15,625s	3125s	625s	125s	25s	5s	1s

Pull out as many 15,625s as you can. How many? 0. So we have

0						
15,625s	3125s	625s	125s	25s	5s	1s

Pull out as many 3125s as you can out of 12,345. How many? 3. We remove these from $12,345$ and get

$$12345 - 3(3125) = 2970.$$

Notice that the remainder should be smaller than the number you were removing (i.e. $2970 < 3125$). So far we have

0	3					
15,625s	3125s	625s	125s	25s	5s	1s

Pull out as many 625s as you can out of 2970. How many? 4. We remove these from 2970 and get $2970 - 4(625) = 470$, and so far we have:

0	3	4				
15,625s	3125s	625s	125s	25s	5s	1s

Pull out as many 125s as you can out of 470. How many? 3. We remove these from 470 and get $470 - 3(125) = 95$, and now we have:

0	3	4	3			
15,625s	3125s	625s	125s	25s	5s	1s

Pull out as many 25s as you can out of 95. How many? 3. We remove these from 95 and get $95 - 3(25) = 20$, and now we have:

0	3	4	3	3		
15,625s	3125s	625s	125s	25s	5s	1s

Pull out as many 5s as you can out of 20. How many? 4. We remove these from 20 and get $20 - 4(5) = 0$, and now we have:

0	3	4	3	3	4	
15,625s	3125s	625s	125s	25s	5s	1s

Pull out as many 1s as you can out of 0. How many? 0.

0	3	4	3	3	4	0
15,625s	3125s	625s	125s	25s	5s	1s

So $12,345$ in base 10 is $343,340$ in base 5, i.e., $343,340_5$.

Notice that in sexagesimal system, a semi-colon is used in place of a decimal point, so $0;30$ becomes $\frac{30}{60}$ in base 10.

Homework for Chapters 1 - 3

All problems will be discussed in class, and all problems with an $(*)$ are due on Thursday, January 22nd. Remember that you may be called on to present any of the problems below, so you should have them all completed for class on Thursday, January 22nd.

1. Express $(3012)_5$ in base 8.
2. For what base is $3 \times 3 = 10$? For what base is $3 \times 3 = 11$?
3. $(*)$ Can 27 represent an even number in some scale? Can 37? Can 72? Explain.
4. $(*)$ Find a base b such that $79 = (142)_b$.
5. An old Babylonian geometry problem is the following, found on the Strassburg tablet of about 1800 BC: "An area A consisting of the sum of two squares is 1000. The side of one square is 10 less than $\frac{2}{3}$ of the side of the other square. What are the sides of the square?" Solve this problem.
6. The Babylonians use the sexagesimal number system (i.e., base 60). Another old Babylonian problem is the following: "A ladder of length $0;30$ is standing upright against a wall. How far does the lower end of the ladder move from the wall if the upper end slides down the wall a distance of $0;6$?" Solve this problem.

   The following three problems are about unit fractions:

7. Show that $\frac{z}{pq} = \frac{1}{pr} + \frac{1}{qr}$ where $r = \frac{p+q}{z}$. This method for finding possible decompositions of a fraction into 2 unit fractions is indicated on a papyrus written in Greek probably sometime between AD 500 and 800, found at Akhmim, a city on the Nile River.
8. $(*)$ Represent $\frac{2}{99}$ as the sum of two different unit fractions in 3 different ways.
9. $(*)$ Show that if n is a multiple of 3, then $\frac{2}{n}$ can be broken into a sum of two unit fractions, of which one is $\frac{1}{2n}$.
10. In the Rhind papyrus, the area of a circle is repeatedly taken as equal to that of the square of $\frac{8}{9}$ of the diameter. This leads to what value for π?

Rule of False Position Example:

Try to solve

$$x + \frac{x}{2} = 21$$

Take a guess: If $x = 2$ then

$$2 + \frac{2}{2} = 2 + 1 = 3$$

Since this needs to be multiplied by 7 to get 21, the solution also needs to be multiplied by 7, so $x = 2 \cdot 7 = 14$ is a solution.

Therefore,

$$14 + \frac{14}{2} = 14 + 7 = 21$$

11. (∗) Solve the following problem from the Rhind papyrus: "A quantity, its $\frac{2}{3}$, its $\frac{1}{2}$, and its $\frac{1}{7}$, added together, become 33. What is the quantity. Solve this problem directly and by using the rule of false position.