Thomas Simpson, The Weaver Mathematician

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Abstract

Thomas Simpson, August 20, 1710 - May 14, 1761, was a British mathematician, inventor and so-called creator of Simpson's rule to approximate definite integrals. Simpson worked as a weaver before indulging in the world of mathematics where his interests were first sparked by the solar eclipse in 1724. Thomas was a member of the Spitalfield's Mathematical Society, being one of forty-nine members in 1736. He then later moved to the Royal Military Academy in Woolwich in 1743, where he was named the Head of Mathematics. Simpson's effect on the world of mathematics in the areas of fluxions, Simpson's Rule, and probability theory have changed the way we see mathematics today.

1 Introduction

Thomas Simpson was born in Leicestershire on August 20, 1710. He was born the son of a weaver who did not care much for Thomas' literacy. Simpson moved away around the age of fourteen and settled in a lodging house with Mrs. Swinfield. Much of Simpson's reason for learning mathematics and astronomy can be contributed to a solar eclipse in 1724. He continued to seek after educating himself and worked as a weaver. Simpson soon began to borrow books from a neighbor on arithmetic and astrology. He became quite a master in both subjects and was known as a fortune teller. Thomas then married Mrs. Swinfield whom was thirty years his senior and later had two children. He turned fortune-telling into a profession to make money, until "an unfortunate accident" in fortune-telling caused Simpson to leave his family and move to Derby, where he resumed weaving.[**?**] Thomas then became a member of the Spitalfield's Mathematical Society, being one of forty-nine members in 1736. "This Society operated as a working men's club and we know that it was a natural choice for a weaver who taught mathematics since of the members by 1744, '- about half were weavers, and the rest were typically brewers, braziers, bakers, bricklayers.'" [**?**] He later moved to the Royal Military Academy in Woolwich in 1743, where he was named the Head of Mathematics. Simpson's effect on mathematics today, although questioned upon originality, has been tremendous. The areas of fluxions, Simpson's Rule, and probability theory have changed the way we see mathematics today which have all been immensely contributed to Thomas Simpson. Most of Simpson's writings are under much controversy and under review of plagiarism. We will further discuss the life and contributions that Simpson made to mathematics.

2 Weaving His Way into Mathematics

Thomas Simpson, who had very little formal education, became a skilled weaver at a very young age. His father was a weaver for years, so naturally Thomas adapted to this profession. By definition, weaving is the action of creating cloth by interweaving two layers of thread usually at ninety degree angles to each other. Weaving is an art that deals with dimensions, area, and shaded regions. This art opened Simpson's eyes to the bisection and trisection of ninety degree angles that would soon become so evident in his mathematics. Many weavers of this time would soon become mathematicians because they found a connection between the threads and angles. This allowed Thomas to solve mathematical problems without the means of a formal education. He would work on weaving as though he was at school solving equations, while still staying true to his fathers profession.

2.1 Mathematician or Fortune-Teller

A peddler wandering through the town of Nuneaton loaned Simpson books on mathematics and astrology, which Simpson studied intently. Oddly enough, a solar eclipse in 1724 really caused Simpson to seek after answers in mathematics. He saw the ways that the sun had been covered by each line of darkness, and wanted to find a way to construct a means of determining the area of this shaded region. He wanted to find how to measure the amount of sun covered, at what times the shaded area increased, and if the eclipse could be broken into halves, thirds, or fourths. [**?**] "It had became customary among astronomers to use the mean of several observations to estimate the true value (covered by the sun in a solar eclipse) but an adequate theoretical background for this procedure was missing, until Simpson." [**?**] He had a divine way of picking up on these mathematical equations because he had been a skilled weaver. Since Simpson had been calculating angles and finding ways to bisect and trisect these angles within weaving without even knowing it since birth, he caught on to mathematics unbelievably quickly. He acquired a reputation as a fortuneteller or astrologer. People believed that he had a divine sense that allowed him to predict the ways of astronomy, and the way the future would prevail. They believed that the devil had cursed Simpson with being able to foretell the future in the darkness of the solar eclipse. He was forced to leave the town. He studied closely and joined many organizations that supported his desire to learn. He later gave up his weaving and became an usher at a school, and by constant and laborious efforts improved his mathematical education, so that by 1735 he was able to solve several questions which had been recently proposed and which involved calculus.[**?**] He perfected De Moivre's (who will later be discussed in further detail) problem of assurance where derivation and integration must be used in full. Simpson took equations that others had previously been introduced and perfected them to bring clarity.

3 Simpson's Rule

Simpson's rule is a method for approximating definite integrals. It works in a similar way to the trapezoidal rule except that the integrand is approximated to be a quadratic rather than a straight line within each subinterval. [**?**] Therefore, we partition the interval $[a, b]$ into n subintervals of equal length $h = \Delta x = \frac{(b-a)}{n}$, with n having to be an even number. A parabola usually passes through three consecutive points $(x_{i-1}, y_{i-1}), (x_i, y_i)$, and (x_{i+1}, y_{i+1}) on the curve. The parabola has equation

$$y = Ax^2 + Bx + C,$$

so the area under it from $x = -h$ to $x = h$ is

$$A_p = \int_{-h}^{h} (Ax^2 + Bx + C)dx$$

$$= \frac{Ax^3}{3} + \frac{Bx^2}{2} + Cx$$

(where we are plugging in -h, and h)

$$= \frac{2Ah^3}{3} + 2Ch = \frac{h}{3}(2Ah^2 + 6C).$$

Since the passes through our three points $(-h, y_0), (0, y_1)$, and (h, y_2), we also have

$$y_0 - Ah^2 - Bh + C, \qquad y_1 = C, \qquad y_2 = Ah^2 + Bh + C,$$

from which we get

$$C = y_1,$$
$$Ah^2 - Bh = y_0 - y_1,$$
$$Ah^2 + Bh = y_2 - y_1,$$
$$2Ah^2 = y_0 + y_2 - 2y_1.$$

We are expressing our area A_p in terms of y_0, y_1, and y_2, and get

$$A_p = \frac{h}{3}(2Ah^2 + 6C) = \frac{h}{3}((y_0 + y_2 - 2y_1) = \frac{h}{3}(y_0 + 4y_1 + y_2).$$

Therefore the area under the parabola through our points $(x_0, y_0), (x_1, y_1)$, and (x_2, y_2), through the shaded region is

$$\frac{h}{3}(y_0 + 4y_1 + y_2).$$

And through points $(x_2, y_2), (x_3, y_3)$, and (x_4, y_4), the area is

$$\frac{h}{3}(y_2 + 4y_3 + y_4).$$

After computing all of the areas under all of the parabolas and summing the approximations, we come up with Simpson's Rule which is stated as:

To approximate $\int_a^b f(x)dx$, use

$$S = \frac{\Delta x}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + ... + 2y_{n-2} + 4y_{n-1} + y_n).$$

The y values are f at the partition points

$$x_0 = a, x_1$$

$$x_0 = a + x, x_2$$

$$x_0 = a + 2x, ..., x_{n-1}$$

$$x_0 = a + (n-1)x, x_n$$

$$x_0 = b.$$

The number n is even, and

$$\Delta x = \frac{(b-a)}{n}.$$

If a function is highly oscillatory, or it lacks derivatives at certain points then they usually have very poor results with Simpson's rule. Therefore we break up the interval $[a, b]$ into a number of small subintervals. Simpson's rule is then applied to each subinterval; the results then produce an approximation for the integral over the entire interval. [**?**]

3.1 The Error with Simpson's Rule

The Error for Simpson's Rule states: "If f(4) is continuous and M is any upper bound for the values of $|f^4|$ on $[a, b]$, then the error E_s in the Simpson's Rule approximation of the integral of f from a to b for n steps satisfies the inequality

$$|E_s| < \frac{M(b-a)^5}{180n^4}.\text{"} [\textbf{?}]$$

We start with a result that says that if the fourth derivative $f^{(4)}$ is continuous, then

$$\int_b^a f(x)dx = S - \frac{b-a}{180} \cdot f^{(4)}(c)(\Delta x)^4$$

and at some point c, between a and b, x approaches zero, and the error,

$$E_S = -\frac{b-a}{180} \cdot f^{(4)}(c)(\Delta x)^4,$$

therefore zero is Δx^4. The inequality

$$|E_S| < -\frac{b-a}{180} max|f^{(4)}(x)|(\Delta x)^4$$

where the maximum is taken on the interval from $[a, b]$, and gives an upper bound for the magnitude of the error. When the magnitude of the second derivative of f, we cannot find the exact value of max $|f^{(4)}(x)|$ and have to replace it with an upper bound. So if M is any upper bound for the values of $|f^{(4)}|$ on $[a, b]$, then

$$|E_s| < \frac{(b-a)}{180} M(\Delta x)^4.$$

We then substitute $\frac{(b-a)}{n}$ for Δx and we get

$$|E_s| < \frac{M(b-a)^5}{180n^4}.$$ [?]

4 Probability Theory

This part of mathematics is concerned with the analysis of random phenomena that much of Simpson's life was dedicated to. He dealt with the probability theory of errors in measurement, dealing with dice, and finding the distribution of mean error. Although an individual coin toss or the roll of a die is random event, if repeated many times the sequence of random events will exhibit certain statistical patterns, which can be studied and predicted. [?] He was on of the first to realize that the chance event is uninfluenced by the events which have gone before. "If a true die has not shown six for thirty throws, the probability of a six is still one sixth on the thirty-first throw." [?] Simpson found this to be true by deriving several equations with an end result of

$$p = \frac{(1+i)a_x}{a_x + 1}$$

He used several proofs and De Moivre's work to compile to this answer. This 'branch' of Simpson's work in mathematics deals with things such as mortality rates and life insurance.

4.1 De Moivre

Simpson worked on the Theory of Errors and aimed to prove that the arithmetic mean was better than a single observation. Thomas Simpson wrote the book, *The Nature and Laws of Chance. The Whole After a new, general, and conspicuous Manner, And illustrated with A great Variety of Examples* in 1740. Unfortunately there was nothing 'new, general, or conspicuous' about it, it was simply a plagiarism of the mathematical parts of the *The Doctrine of Chances, First Edition* which was introduced by De Moivre.[?] Then again based upon *Annuities upon Lives*, by De Moivre, Simpson published another book named *The Doctrine of Annuities and Reversions, Deduced from General and Evident Principles : With Useful Tables, Shewing the Values of Single and Joint Lives, etc. at different Rates of Interest.* Simpson basically took the book that De Moivre wrote, and published his own with revisions to make this work better. "He made three important new contributions (1) a life table based on the London bills of mortality, (2) age based on this life table; and (3) rules for calculating joint-life annuities for different ages from the tabulated joint-life annuities." [?] Simpson reduced the size of text from De Moivre's two hundred and fifty six page book to a mere eighty five page book leaving out discussions on concrete games of chance. De Moivre then came out with the Second Edition of the book he had previously written. He ended the preface with a few remarks of his feelings on the stolen work of Simpson by saying: "After the pains I have taken to perfect this Second Edition, it may happen, that a certain Person, whom I need not name, our of Compassion to the Public, will publish a Second Edition of his Book on the same subject, which he will afford at a very moderate Price, not regarding whether he mutilates my Propositions, obscures what is clear, makes a Shew of new Rules, and works by mine; in short confounds in his usual way, ever thing with a croud of useless Symbols; if this be the Case, I must forgive the indigent Author and his disappointed Bookseller." [?] Simpson then published a sixteen page "Appendix, containing Some Remarks on Mr. Demoivre's Book on the same Subject, with Answers to Some Personal and Malignant Misrepresentations, in the Preface thereoff "..." to clear myself from a charge so highly injurious, and do justice to the foregoing work" to clear his name of plagiarism. [?] Simpson not only defended his work as original but as insightful. Simpson then ended the Appendix by saying, "Lastly, I appeal to all mankind, whether in his treatment of me, he has not discovered an air of self-sufficiency, ill-nature, and inveteracy, unbecoming a gentleman." [?] De Moivre did not respond to Simpson's remarks, but then later removed the criticisms of Simpson from the preface in following editions. Most of the work that is noted in Simpsons name has many stipulations. De Moivre's however was not the only work that there was controversy over plagiarism. The First Master at the Military Academy's books on geometry and the previous editor of the *LadiesDiary*, all had pressing issues with the statements that Simpson had in later books. Although he has become a noted name in the world of calculus, his name is hardly ever mentioned without De Moivre's work. Together De Moivre and Simpson proved tons of work concerning probability and life tables. Without the work of De Moivre, there is no telling how far Simpson would have come to revolutionize mathematics in the manner that he did. Although they both had a deep revulsion for the other, they were both very dependant on each other.

5 Conclusion

Thomas Simpson, although questioned about originality, has contributed to mathematics in numerous ways. His teaching , writing, and editorial work resulted in a large correspondence about mathematical problems such as the theory of fluxions, laws of chance, annuities and revisions, algebra, geometry, physical astronomy, and speculative mathematics." [?] Although born the son of a weaver, Thomas tried to perfect a world of mathematics that has change the way we use calculus and probability today. From weaving to foretelling the future to now a noted mathematician, Thomas came a long way to simplify the work of others for more practical use.

References

[1] Moivre, A. de (1718). *The Doctrine of Chances: or, A Method of Calculating the Probability of Events in Play.* Peason, London.

[2] Thomas, George B. *Thomas' Calculus Early Transcendentals*, Weir. Hass. Giordano. (2006), 592-597.

[3] Gale, Thomas. *World of Mathematics on Thomas Simpson*, (2005-2006). $www.bookrags.com/biography/thomas-simpson-wom/$ [Online; accessed 02-April-2007]

[4] Hald, Anders. *A history of probability and statistics and their applications before 1750*, (1913), 214-567

[5] Simpson, T. (1742) *The Doctrine of Annuities and Reversions, Deduced from General and Evident Principles : With Useful Tables, Shewing the Values of Single and Joint Lives, etc. at different Rates of Interest*. Nourse, London. 2nd Ed., 1775.