Euclid, The Geometer

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Abstract

Euclid of Alexandria was a prominent mathematician living in Alexandria, Egypt around 300 B.C.. He produced many books on his studies in mathematics. One in particular was the book of *Elements* which was written to be an instructional geometry book but become one of the most talked about books in mathematics still today.

1 Intro

Euclid of Alexandria can be considered a pioneer in the field of mathematics. Although his name is not well known to the general population, those that have studied the history of mathematics have likely run across his name. The only things known about Euclid derive from a Greek philosopher named Proclus. Proclus was a student and later a teacher at Plato's Academy. He had access to many books while at Plato's Academy and that is where he wrote *Commentary on Euclid*, which is are major source of Euclid's geometry and his life. Euclid is still a mystery to us all. No one has any proof when and were he lived. We do know that he had written several books which gives us insight on where he may have studied. Some of his predecessors works also give us an idea on the time that he flourished. Euclid had many books that ranged from astrological to medical. He tied mathematics in to each of these topics. The subject of astronomy was covered in the book of *Phaenomena* which contains the application of spherical geometry. You can compare the book of *Optics* to the medical field as he was trying to prove how people see. This paper reviews Euclid's life, accomplishments, and the influence he had on the profession of mathematics.

2 Life

Euclid's existence is unclear; the time and place of both his birth and death still baffles historians today. There is much confusion surrounding the name Euclid, because of a Socratic philosopher named Euclid of Megara which lived around 400 B.C. During the Middle Ages he was mistaken for Euclid of Alexandria. A quote from Proclus found in an article in the *History of Mathematics* states that "Not much younger than these (sc. Hermotimus of Colophon and Philippus of Medma) is Euclid, who put together the *Elements*, collecting many of Eudoxus' theorems, perfecting many of Theaeteus', and also bringing to irrefragable demonstrations the things which were only somewhat loosely proved by his predecessors. This man lived in the time of the first Ptolemy. For Archimedes, who came immediately after the first Ptolemy, makes mention of Euclid... He is then younger than the pupils of Plato but older than Eratosthenes and Archimedes; for the latter were contemporary with one another, as Eratosthenes somewhere says."[2] This helps us figure out the time period in which Euclid was alive. Euclid lived during the reign of Ptolemy I, which was from 306-283 B.C., he was younger than Plato who lived between 428-347 B.C., and older than Archimedes and Eratosthenes, whom lived between 287-212 B.C. and 276-194 B.C. respectively. It is concluded that Euclid flourished around 300 B.C. Euclid's place of birth is unknown as well. He was most likely born in Greece, but we are still unsure of the town. However it is known that he went to Athens to study at Plato's Academy. Proclus said that Euclid "belonged to the persuasion of Plato"[3]. A theory of the five regular solids was taught by Plato at the Academy and was later used by Euclid in his book the *Elements*. This brings us to the assumption that Euclid was in Athens at the time of Plato's pupils.

Euclid was known to be a humorous person. Proclus tells us about Euclid joking around with the ruler of Alexandria saying: "...Archimedes makes mention of Euclid: and, further, they say that Ptolemy once asked him if there was any shorter way than that of the *Elements*, and he answered that there was no royal road to geometry..."[2]. There was also an occurrence when "Euclid was asked by one of his students, which had just learned a new theorem, what he should gain from learning such things, Euclid tells his slave. give him three pence, since he must make gain out of what he learns."[4]. From these quotes we can conclude that Euclid had a sense of humor, laid back, and impartial. He would joke with people ranging from one of his students to the ruler of Alexandria.

Euclid and a man named Demetrius Phalereus were invited by the First Ptolemy to open a school in a museum and to run the Library at Alexandria. The Library at Alexandria was the largest library in the world at that time, containing over 600,000 papyrus rolls. This is clear from the remark of Pappus about Apollonius: "he spent a very long time with the pupils of Euclid at Alexandria, and it was thus that he acquired such a scientific habit of thought." [4] which leads us to conclude that Euclid taught at and founded the school in Alexandria. A historian, Itard, who wrote *Les livres arithmetique d'Euclide*, researched Euclid and proposed three hypotheses, since there is very little known about Euclid. The first: "Euclid was the leader of a team of mathematicians working at Alexandria. They all contributed to writing the 'complete works of Euclid', even continuing to write books under Euclid's name after his death."[2] Which could be true since he taught and founded a school there. The second: "Euclid was not a historical character. The 'complete works of Euclid' were written by a team of mathematicians at Alexandria who took the name Euclid from the historical character Euclid of Megara, who had lived about 100 years earlier,"[2] which is probably the most fictitious of them all. "Nevertheless the 20th century example of Bourbaki shows that it is far from impossible. Henri Cartan, Andre Weil, Jean Dieudonne, Claude Chevalley, and Alexander Grothendieck wrote collectively under the name of Bourbaki and Bourbaki's *Elements de mathematiques* contains more than 30 volumes."[2]. This makes people think that Euclid may not even be real. Some people could believe this because of the fact that not much is known about Euclid, and with this other information about Bourbaki can make people to believe that Euclid is a just a made up name. There is just not enough hard evidence to prove that he was real or not. The last hypothesis and the one assumed for this paper is, "Euclid was an historical character who wrote the *Elements* and the other works attributed to him."[2] Euclid seems to have lived the rest of his life in Alexandria where he wrote the book of the *Elements* which was used as the geometry book and teachings in the west for over 2,000 years.

3 Other Works

Other than Euclid's *Elements*, which he is most known for, there were some other books that he completed that were quite impressive. Some of them are the *Data, On Divisions of Figures, Phaenoma,* and *Optics*. "All survived in the original Greek except *Divisions*, which is only partially preserved in Arabic. These works all follow the basic logical structure of the *Elements*, having definitions and long extensive propositions."[5] The book *Data* contains 96 propositions, proofs, and elementary geometry. "Its form is that of a proposition proving that, if certain things in a figure are given (in magnitude, species, etc.), something else is given."[5] which follows the same form as the *Elements*. "The book, *On Divisions (of Figures)* has been lost in Greek, but has been discovered in Arabic. The divisions are divisions into figures of the same kinds as the original figures."[5] It consisted of 36 propositions and like proofs of dividing one triangle into two. The *Phaenomena* book has 18 propositions and is what we would call today, applied mathematics; it is about the geometry of spheres applicable to astronomy.[14] Euclid did not only study mathematics but used his knowledge in geometry to write a book in astronomy as well.

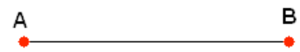
Euclid was also given credit for the following works, *Conics, Porisms, Pseudaria (Book of fallacies)*, and *Surface Loci*, but were later lost, and we have no proof of who completed them. The book of *Conics*, "was a work on conic sections that was later extended by Apollonius of Perga into his famous work on the subject."[8] The work *Conics*, contained a total of four books which Apollonius extended to an extra four books to make it a total of eight. *Porisms* was another which "might have been an outgrowth of Euclid's work with conic sections, but the exact meaning of the title is controversial"[8]. The *Book of Fallacies*, or *Pseudaria*, "was an elementary text about errors in reasoning."[8] The book showed beginners how to avoid errors with reasoning by setting something that is correct next to incorrect, so they would know what to do and what not to do. "*Surface Loci* concerned either loci (sets of points) on surfaces or loci which were themselves surfaces; under the latter interpolation, it has been hypothesized that the work might have dealt with quadric surfaces."[8] It contained a total of 2 books.

4 The Elements

Other than the *Bible*, The *Elements* has been printed more times than any other piece of literature. The *Elements* was Euclid's most noted masterpiece. "The first printed edition of the *Elements* was made at Venice in 1482 and contained Campanus' translation." [10] No work, except the Bible, has been more widely used, edited, or studied, and probably no work has exercised a greater influence on scientific thinking. Over one thousand editions of Euclids Elements have appeared since the first one was printed.
citeEves The *Elements* was a rare book for back then. The *Elements* was a rare book for back then and was used in the geometry teachings in the west for many years. The *Elements* contained thirteen books, which consisted of definitions, postulates, axioms, and in depth proofs. The first book of the *Elements* starts off with 23 definitions, 5 postulates,

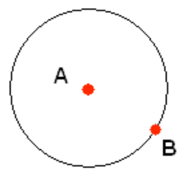
5 axioms, and 48 propositions. The five postulates and five axioms was the base of the *Elements*. A postulate is something taken as self-evident or assumed without proof as a basis for reasoning; a proposition that requires no proof, being self-evident, or that is for a specific purpose assumed true, and that is used in the proof of other propositions; axiom, based on *Websters.com*. The five postulates with which Euclid started his *Elements* of were:

1. To draw a straight line from any point to any point.

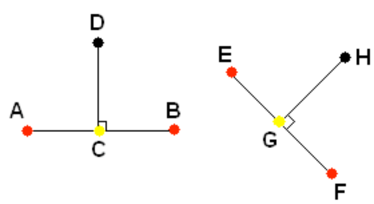


2. To produce a finite straight line continuously in a straight line.

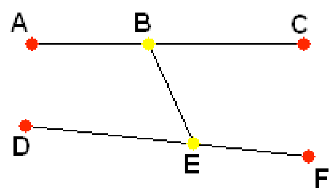
3. To describe a circle with any center and distance.



4. That all right angles are equal to one another.



5. That, if a straight line falling on two straight lines, if produced in-definitely, meet on that side on which are the angles less than the two right angles



The first three postulates describe points, lines, and circles. The fourth postulate was that all right angles are congruent and fifth postulate cannot be proven as a theorem. “In 1823, Janos Bolyai and Nicolai Lobachevsky independently realized that entirely self-consistent ”non-Euclidean geometries” could be created in which the parallel postulate did not hold.”[13] There was also the axioms that are defined as a self-evident truth that requires no proof or a proposition that is assumed without proof for the sake of studying the consequences that follow from it. These are the axioms that Euclid uses:

1. Things which are equal to the same thing are also equal to one another.
2. If equals be added to equals, the wholes are equal.
3. If equals be subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another
5. The whole is greater than the part .

An example for the first axiom would be if $a = b$ and $b = c$, then $a = c$. Also for the second axiom an example would be if $a = b$ and $d = c$, then $a + c = b + d$. The third axiom is similar to the second in the sense of $a = b$ and $d = c$, then $a - c = b - d$. Euclid called the axioms common notions. Other people added more axioms, but these were the original five that Euclid used in the *Elements*. In the *Elements*, Books 1-6 contain plane geometry; Books 7-9 cover number theory; Book 10 has the theory of irrational numbers, and Books 11-13 entail three-dimensional geometry. Because of the great detail and many proofs and propositions in all thirteen books; This paper will only address one proposition from each of them. Here is an overview of the 13 books of the *Elements*.

Book	Definitions	Theorems	Problems	Porisms	Lemmas	Includes
I	23	34	14	1	-	Basic plane geometry
II	2	12	2	-	-	Geometric algebra
III	11	31	6	1	-	Circles and angles
IV	7	-	16	1	-	Construction of regular polygons
V	18	25	-	2	-	Abstract algebra
VI	3	23	10	3	-	Similar figures and geometric proportions
VII	22	33	6	1	-	Basic number theory
VIII	-	25	2	1	-	Continued proportions in number theory
IX	-	36	-	1	-	Number theory
X	16	91	24	4	11	Classification of irrational numbers
XI	28	34	5	1	1	Basic solid geometry
XII	-	16	2	2	2	Measurement of solids
XIII		12	6	1	3	Constructing regular polyhedra
Total	120	372	93	19	16	

4.1 Book I

Book I of the *Elements* is on basic plane geometry. As stated before, it contains 23 definitions, 5 postulates, 5 axioms, and 48 propositions. In the first proposition it states, "To construct an equilateral triangle on a given finite straight line,"[6] First you draw a line AB. Then using a compass with the center at A you draw the circle BCD and radius AB. Also set the compass and draw another circle with the center at B producing the circle ACE. Now since C is on the circle BCD and AB is the radius, then $AC = AB$. Same for the circle ACE with the radius BA; since C is on the circle ACE, then $BA = CB$. From the first axiom we have proven that $AC = AB = BA = CB$. This gives us an equilateral triangle when we on a given finite straight line.

4.2 Book VII

Book VII consists of basic number theory. In proposition two, he states, "To find the greatest common measure of 2 given numbers not relatively prime." [6] This gives us what we call the greatest common divisor (gcd). "Let AB and CD be the two given numbers not relatively prime. If now CD measures AB, since it also measures itself, then CD is a common measure of CD and AB." [6] If it does not then, "when the less of the numbers AB and CD being continually subtracted from the greater, some number is left which measures the one before it. Now let CD, measuring BE. Leave EA less than itself. Let EA measuring DF, leave FC less than itself, and let CF measure AE. Since CF measures AE, and AE measures DF. therefore CF also measures DF. But it measures itself, therefore it measures the whole CD. But CD measures BE, therefore CF also measures AB and CD. Therefore CF is a common measure of AB and CD."[6] This is commonly known as Euclid's algorithm for rational numbers. In terms that are a little easier to comprehend we let $a = bm + r$ and are trying to find a number x which divides both and a and b. The number x will also divide r since:

$$r = a - bm = sx - mtx = (s - mt)x$$

, where $a = sx$ and $b = tx$. We can find a number v which divides b and r. ($b = s|v$ and $r = t|v$), which in this case the number v will also divide a since:

$$a = bm + r = s|vm + t|v = (s|m + t|)v$$

. This says that every common divisor of a and b is a common divisor of b and r.

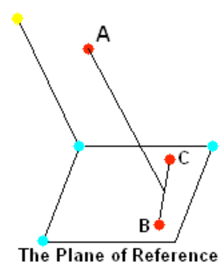
4.3 Book X

Book X, proposition 15, claims, "If two commensurable magnitudes are added together, then the whole is also commensurable with one of them; and, if the whole is commensurable with one of them, then the original magnitudes are also commensurable."[6] First of all the word commensurable is defined as, exactly divisible by the same unit an integral number of times; Used of two quantities. [12] "Let the two commensurable magnitudes AB and BC be added together. The whole AC is also commensurable with each of the magnitudes AB and BC. Since AB and BC are commensurable, some magnitude D measures them. Since then D measures AB and BC, therefore it also measures the whole AC. But it measures AB and BC also, therefore D measures AB, BC, and AC. Therefore AC is commensurable with each of the magnitudes AB and BC. Next, let AC be commensurable with AB. I say that AB and BC are also commensurable. Since AC and AB are commensurable, some magnitude Dmeasures them. Since then D measures CA and AB, therefore it also measures the remainder BC. But it measures AB also, therefore D measures AB and BC. Therefore AB and BC are commensurable. Therefore, if two commensurable magnitudes are added together, then the whole is also commensurable with each of them; and, if the whole is commensurable with one of them, then the original magnitudes are also commensurable."[6]

4.4 Book XI

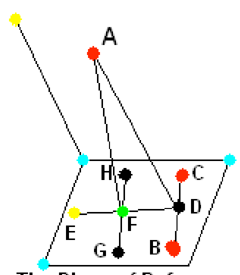
Proposition 11 in Book XI state "To draw a straight line perpendicular to a given plane from a given elevated point."[6] We start to get into the three-dimensional aspect of geometry at the beginning of Book XI. "Let A be the given elevated point, and the plane of reference the given plane. It is required to draw from the point A a straight line perpendicular to the plane of reference. Draw any straight line BC at random in the plane of reference, and draw AD from the point A perpendicular to BC. Then if AD is also perpendicular to the plane of reference, then that which was proposed is done.

Figure 7:



But if not, draw DE from the point D at right angles to BC and in the plane of reference, draw AF from A perpendicular to DE, and draw GH through the point F parallel to BC. Now, since BC is at right angles to each of the straight lines DA and DE, therefore BC is also at right angles to the plane through ED and DA. And GH is parallel to it, but if two straight lines are parallel, and one of them is at right angles to any plane, then the remaining one is also at right angles to the same plane, therefore GH is also at right angles to the plane through ED and DA. And GH is parallel to it, but if two straight lines are parallel, and one of them is at right angles to any plane, then the remaining one is also at right angles to the same plane, therefore GH is also at right angles to the plane through ED and DA. Therefore GH is also at right angles to all the straight lines which meet it and are in the plane through ED and DA. But AF meets it and lies in the plane through ED and DA, therefore GH is at right angles to FA, so that FA is also at right angles to GH. But AF is also at right angles to DE, therefore AF is at right angles to each of the straight lines GH and DE. But if a straight is set up at right angles to two straight lines which cut one another at their intersection point, then it also is at right angles to the plane through them. Therefore FA is at right angles to the plane through ED and GH. But the plane through ED and GH is the plane of reference, therefore AF is at right angles to the plane of reference. Therefore from the given elevated point A the straight line AF has been drawn perpendicular to the plane of reference."[6]

With all of these well constructed books, including extensive proofs, Euclid lead the way for many other "Greats" to proceed him.



The Plane of Reference

5 Euclid's Influences

Figure 9: Euclid and some of his students

Euclid's work became a model in mathematics which assisted Issac Newton in his theory of gravitational and planetary motion. Euclid's work on *Phaenomena* influence Issac Newton in his theory of gravitational and planetary motion. Newton, at first, was having trouble with Euclids Phaenomena, and put it away. Then his teacher at Cambridge University gave him some guidance, and later, Newton, produced *Principia* which stated the three universal laws of motion, gravity, and defined the law of universal gravitation. "Newton's interest in mathematics began in the autumn of 1663 when he bought an astrology book at a fair in Cambridge and found that he could not understand the mathematics in it. Attempting to read a trigonometry book, he found that he lacked knowledge of geometry and so decided to read Barrow's edition of Euclid's Elements. The first few results were so easy that he almost gave up but he:-

... changed his mind when he read that parallelograms upon the same base and between the same parallels are equal.

Returning to the beginning, Newton read the whole book with a new respect."[9] Thomas Jefferson the 3rd president, was also influenced by Euclid. He studied Euclid and Newton while he was at William and Mary University. Thomas Jefferson had written a large part of the Declaration of Independence. In the Jefferson Memorial in Washington, DC there is a sentence that says in the Declaration of Independence that is written up on the wall: "We hold these truths to be self-evident: . . . ". You would only put it like that, you would only express your thoughts in those precise words, if you had studied Euclid at an impressionable age.[11] The United States Military Academy at West Point was established during Jeffersons years as President. Jefferson wanted to establish colleges with strong mathematical foundations. "The other major and equally influential educational institution which Jefferson created, at every level from site and architecture to syllabus and appointments, is, of course, the University of Virginia, in its conception the most enlightened and liberal college of the New World. In his plans for this institution mathematics had a more prominent place than at most American colleges of the period."[11] Euclid also influenced some other well known mathematicians such as Leonardo Fibonacci. Fibonacci wrote the book *Practica Geometriae* (the Practice of Geometry). "It is worth noting that, in 1915, R.C. Archibald based his *Euclid's Book on Division of Figures* (a restoration of Euclid's lost work) on Woepekes's French translation of an Arabian MS, and on Fibonacci's *Practica Geometriae*."[7] Fibonacci did not agree with Euclids way of solving equations by square roots, so he devised his own way and wrote it in (Babylonian) sexagesimal notation. Another, who was influenced by Euclid was an Islamic mathematician known as Alhazen. "A significant observation in the work contradicted the beliefs of many great scientists, such as Ptolemy and Euclid. Alhazen correctly proposed that the eyes passively receive light reflected from objects, rather than emanating light rays themselves. (Wikipedia) Even though Alhazen went against Euclid on this subject, he still was influenced enough to research and write his own hypothesis on the study of vision. There is also Pappus of Alexandria, who was a Greek mathematician. He recorded and enlarged the results of his predecessors.(High Beam Encyclopedia). Pappus lived close to 500 years after Euclid. "He wrote *Mathematical Collection*, and eight book series that included commentary and historical notes, as well as several original propositions and extensions of existing works. In book VII, he discusses 12 treatises of his past which included Euclid's *Surface Loci*."[10] Pappus, being in Alexandria, had access to many works from the greats, like Euclid, that are now lost.

6 Conclusion

His life is a mystery to everyone but his writings are not. All of Euclids books had some kind of influences on later mathematicians. All of us Math majors study geometry from his knowledge and writings from the Elements. You could say that Euclid is one of the greatest geometry teachers, because we have used his book of the Elements to study geometry for many years. He may not have been right on all of his books but the intelligence that he had to have to create the books that were written by him is outstanding. He was able to tie mathematics into different subjects that as an average human being, we would have never dreamt it could be possible. Who would have ever thought that you could tie mathematics in with music? Euclid did. Euclid was not only a brilliant mathematician but a great teacher and fair person. From what we can gather from the information that has been passed down about him; he treated everyone from the king to his students equally. The king was in fact one of his students as well. Ptolemy 1, the king/ruler of Alexandria, knew that Euclid was a brilliant man and wanted him as a teacher and as the person to start up his new university in Alexandria. Euclid did take up his offer and because of that was able to pass down his knowledge to people like you and me. As stated earlier, his life is a mystery to all historians and mathematicians. With all the questions still lingering around Euclid, his life, his works, and even his existence, one has to wonder if we have been tricked by some brilliant ancient mathematicians.

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