

# The Father of Calculus

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## Abstract

Calculus has long been looked upon as advanced mathematics even though it was discovered in ancient Greece. Archimedes was the scholar who became famous for his discovery of it in the middle of the second century B.C. His work inspired other people such as Kepler, Fermat, and Newton to continue the study of calculus. Archimedes started using methods to find what is now today called limits, integrals, and derivatives. Thus in this paper we are going to look at why he is noted as the Father of Calculus.

## 1 Biography

Archimedes was a scholar who lived in Syracuse, Greece. He was born in 287 but the date of his death is still unclear. Greece was in a war and during the end of Archimedes life, the warring king ordered that Archimedes was to not be harmed. During the chaos of the war though, a soldier had stabbed him, thus the date of his death is unclear but is figured to be around 212 or 211 B.C.

During his life he went to study in Alexandria, Egypt for a short time but eventually returned to Syracuse. Little is known of his personal life however; all that is left is his works. We know he studied astronomy, physics, engineering, and extensive geometry. He was made famous for some of his inventions as well, such as his invention of the burning mirrors that was used to pin point the heat of the sun on a point and Archimedes claw that was used to flip ships.

## 2 Idea of Calculus

First to understand what Archimedes accomplished we must take a look at modern calculus. Calculus is divided into three different sections: limits, derivatives, and integrals.

## 2.1 Limits

Limits look at a function and where it is headed. When calculating a limit to infinity you are more or less determining if a function reaches a number or if it continues to a positive or negative infinity. On the other hand a function does not always have a limit. If it is not continuous or if the function comes to a peak then there is no limit. A limit can be calculated at a point by looking at what value the function gives as you approach it from the left and the right. The way to calculate a function as the variables approach infinity can be done the same way.

## 2.2 Derivatives

Derivatives give the slope of a point on a given function. The definition of a derivative comes from limits. It is noted as:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

From derivatives we can find key points in functions as well, such as a maximum or minimum point or even the points where a functions graph changes from a positive to a negative slope.

## 2.3 Integrals

Integrals are a form of calculus that is used to calculate the area under a curve. This form of calculating this comes from derivatives and limits combined. The definition of an integral is as follows:

$$\int f(x)dx = \lim_{n \rightarrow \infty} \sum_{i=1} f(x)\delta x$$

The idea is to take the sum of the limits within a range of two points, and this will give the area within that range.

## 3 Method of Exhaustion

The method of exhaustion was a technique where you calculate the area of a shape by getting the difference in the inscribed and circumscribed areas. The idea was to take the inscribed and circumscribed polygon and have more sides or by adding in more polygons. Taking the difference of the inscribed and the circumscribed polygon would give the area. The idea was to continually find the difference of the areas until the difference was so small that it was no longer relevant to continue with adding in more polygons or adding more sides to your polygons. In the case of the parabola, the method would continually add in more triangles inside until the change in area was minute. For the other cases Archimedes

would alter the polygon that he was using as inscribed and circumscribed to have more sides thus making the actual difference in the area closer and closer to that of the object he was searching for. It can be noted that the further that the method of exhaustion was taken, the less and less difference there was with the answer that was obtained before.

## 4 Infinity

The method of exhaustion brings us to what is known as infinity. If the method was taken continuously, then infinity would be reached. However that being impossible Archimedes noted that at a point he would have gone enough, going any further would not produce any more significant results. Calculating the area under a circle using inscribed and circumscribed would required them be calculated using a polygon of infinite sides. This brought Archimedes to realize that there was such a thing as infinity but was only able to estimate it.

## 5 Area

### 5.1 Parabola

Where Archimedes started to discover and use calculus was in the calculation of the area of a parabola. His idea was to segment a parabola into a quadrant, and then subdivide it into triangles. Thus there is the big triangle in the center, and in the remaining sides of the parabola you make another triangle and continue this.

Using Figure 1 for example, Archimedes was able to calculate the area of the green triangle. Now to get close to the area of the parabola he added in the two blue triangles and added the area of those to that of the green one. He used the same method of for the red triangles and added them to the sum of the other three triangles. Archimedes continued this process in order to approximate the area of the parabola.

### 5.2 Circles

#### 5.2.1 Finding the Area

Following the same pattern of approximation the area of the parabola, Archimedes applied to find the area of a circle. He used hexagons to get the general approximation of the area. Using different sized hexagons he was better able to find the area of the circle. The first hexagon he sized so that the points were touching the circle and thus was inside the circle. The next hexagon he made it so that the midpoints of the sides were touching the circle, thus making another hexagon that was larger than the circle. He used polygons such as the hexagon and an octagon because calculating the area of it could be easily calculated. This is known as inscribed and circumscribed. Then in order to find the area of the circle,

Figure 1: Parabola Exhaustion

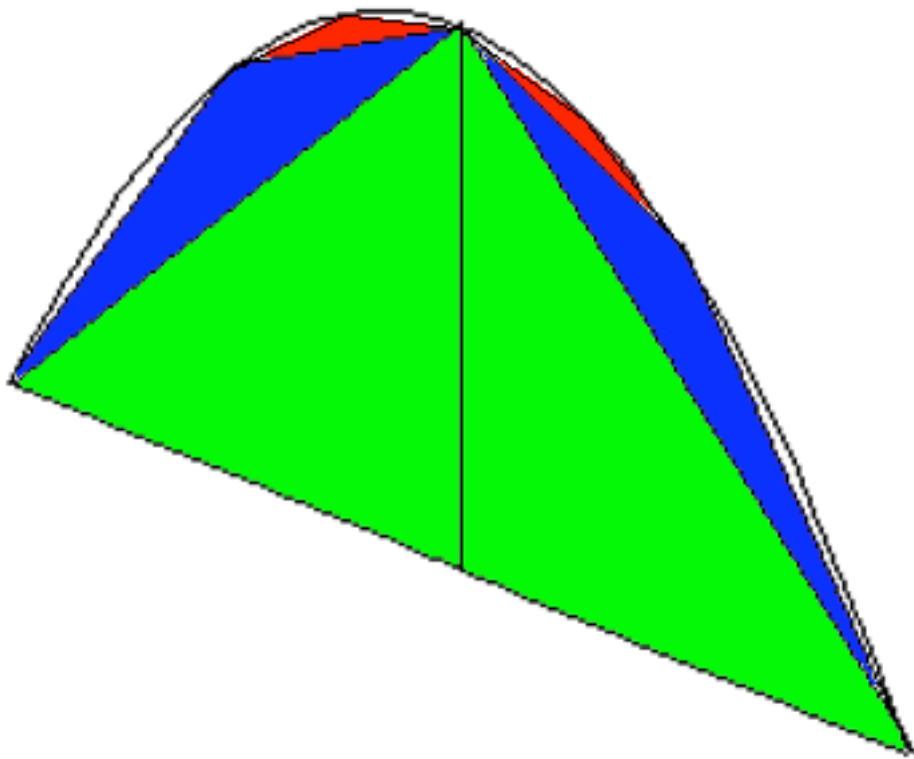
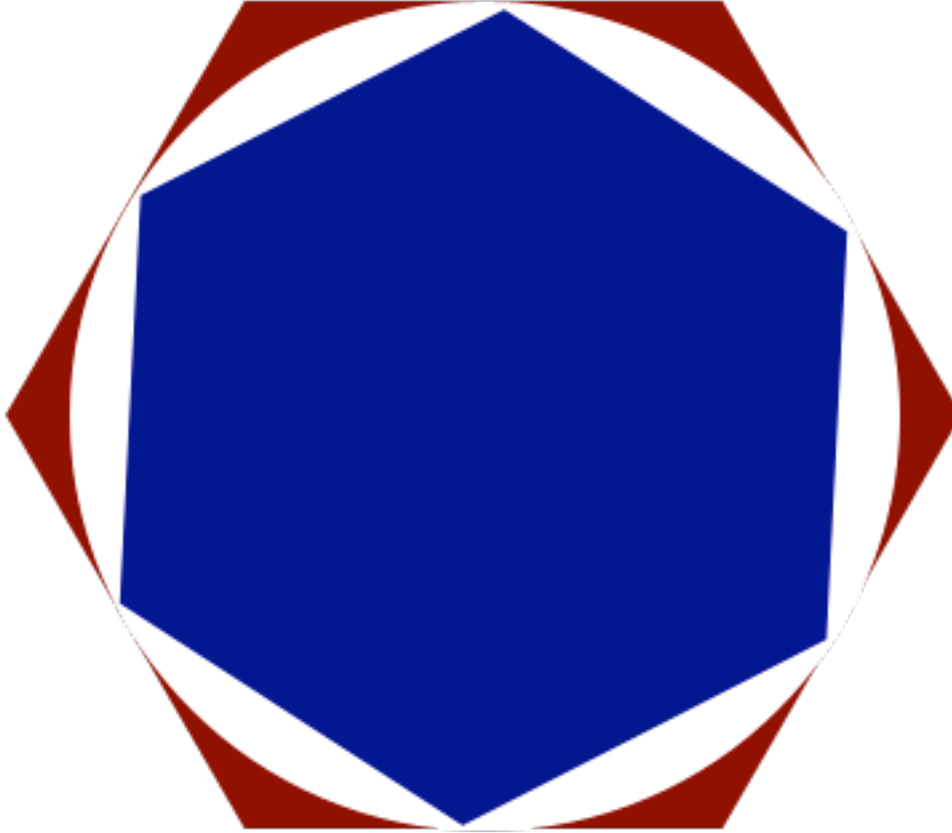


Figure 2: Circumscribed and Inscribed Circle



he took the area of the circumscribed hexagon (the larger hexagon) and subtracted it from the inscribed hexagon (the smaller hexagon that fits into the circle) which gave an approximation of the area.

From this Figure 2, we can see how this was calculated as an example. With the circle being white and the inscribed hexagon being blue and the circumscribed hexagon is in red. The area would be approximated by finding the difference between the two hexagons.

### 5.2.2 $\pi$

While calculating the area of a circle he also came to calculate  $\pi$ . Archimedes applied his method of exhaustion to finding the area of the circles, to do that he add more and

more sides to the inscribed and circumscribed polygon. He eventually came to a polygon that had ninety-six sides, which is relatively close to the circumference of the actual circle. He was able to calculate the perimeter of the inscribed and circumscribed hexagons to approximate the area circumference of the circle. He was able to do this because the difference of the perimeter of the inscribed and circumscribed polygons will be the average of the actual circle. From using method of exhaustion he was able to estimate the area of pi which he found lay between  $3\frac{1}{7}$  and  $3\frac{10}{71}$ . Archimedes approximation of  $\pi$  was only 0.00126 and 0.000748 off of the actual value of  $\pi$ .

### 5.3 Three-dimensional space

Archimedes love for geometry extended beyond the two-dimensional space. He continued his calculation of area into three-dimensional space. He applied his same method of exhaustion using objects that he could easily calculate the volume of to approximate the area a sphere and other hard to calculate three-dimensional objects. He was able to again use his method of exhaustion of easy to calculate objects to find the surface area of other objects.

Archimedes' most famous discovery from the three-dimensional object was that of the sphere in the cylinder. Using his method of exhaustion in three-dimension space he was able to find the surface area and the volume of a sphere in relation to the cylinder. He was so proud of this finding that he had it put on his tombstone.

## 6 Archimedes the Father

Calculus today still holds to the techniques that Archimedes used. The usage of finding the difference of the inscribed and circumscribed polygons is still used today as a foundation for learning calculus. His method of exhaustion was the first was to calculate the area under a curve, which leads into the discovery of integrals. Through his work using geometry and working with the basis of finding the area under a curve he also stumbled onto limits which. The limits are part of derivatives which are a part of integration. So really, Archimedes discovered the basics of calculus through his work of geometry. Being the first to find this and work with it, he was noted the father of calculus.

## 7 Conclusion

Since Archimedes, calculus has evolved into more complex techniques. However these techniques for calculating the limits, derivatives, and integrals have become a general study. Mathematicians have spent their entire lives to the study of calculus proving and expanding on the work that he started. Archimedes discovered calculus by trying to find the area

of different objects and shapes. Even though he did not declare it as calculus, he is the founding father of it.

## References

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