Renaissance Mathematics

I. Abstract

This research paper will discuss the mathematics during the time of the renaissance period. It will start with the French business schools called the abacists, then a few of the mathematicians of the period then finish with the solution of the cubic.

II. Introduction

The period of the Renaissance is defined as

> The transitional movement in Europe between medieval and modern times beginning in the 14th century in Italy, lasting into the 17th century, and marked by a humanistic revival of classical influence expressed in a flowering of the arts and literature and by the beginnings of modern science. [8]

Mathematics, however, tended to lag behind the other sciences in the period of the Renaissance. This is mostly due to the economic hardships and other social issues that plagued Europe of this time. But as the need arose for higher mathematics, many great achievements were realized in the subject during this period [3].

During the early years of the Renaissance, the scholars of Western Europe had little interest in mathematics and most universities had not changed their mathematics departments in over two hundred years. Also, the Black Plague and a recession terrorized the peoples during the fourteenth and half of the fifteenth centuries. These hardships combined with a war between England and France from 1337 to 1443 put mathematics on the back burner for intellects of the time. Also mathematics was "seen as irrelevant and unimportant [3]."

III. Abacists

After the crusades, Italian merchants became more stationary, not needing to travel to sell their goods. This allowed for business deals. The merchants had more time to stay home in Italy and send their employees out to make their exchanges. These business deals opened up a whole new need for mathematics such as financial capitalization, credit instruments, and a way to compute compound interest (Allan). The need for these new mathematical skills also meant that there was a need for someone to teach these skills. Thus the Italian abacists were created.

The abacists were schools created to teach business mathematics. The majority of their students were the sons of merchants [7]. The abacists were instrumental in teaching the "new" Hindu-Arabic decimal system that we use today and the algorithms for using it [5]. There were many who resisted the movement to the new system despite its obvious advantages. The biggest advantage being that the old system of using a counting board required not only a board, but a bag of counters to be carried around, with the new system, only a pen and paper was needed [5].

With the new Hindu-Arabic number system in place, the abacists could teach their students problem solving techniques using arithmetic and Islamic algebra to solve business problems that their students would have to use when they joined their fathers' companies. In addition to the business problems, the students were also taught recreational problems which dealt with algebra, geometry, elementary number theory, the calendar, and astrology. The text books of these schools had many examples with elaborate solutions. Some examples of these problems are:

- The gold florin is worth 5 *lire*, 12 *soldi*, 6 *denarii* in Lucca. How much (in terms of gold florins) are 13 *soldi,* 9 *denarii* worth? (One needs to know that 20 *soldi* make up 1 *lira* and 12 *denarii* make 1 soldo.)
- The *lira* earns 3 *denarii* a month in interest. How much will 60 *lire* earn in 8 months?
- A field is 150 feet long. A dog stands at one corner and a hare at the other. The dog leaps 9 feet while the hare leaps 7. In how many feet and leaps will the dog catch the hare?

Even though the problems that were solved in the abacists schools were quite simple, they instilled the basic algebra skills upon its students that were needed for any future advances. These further advances were to be using abbreviations and symbols rather than words to describe operations and unknowns leading to the modern mathematical equation we think of today [5].

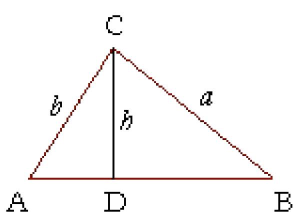
IV. Mathematicians

The period of the Renaissance opened the doors for many individual mathematicians to make their own mark on history. Most of them contributed in the transition from the mathematical problems of old into a more standardized equation type math.

V. Johann Müller Regiomontanus

Johann Müller Regiomontanus (1436 – 1476) wrote a book in 1464 called *De triangulis omnimodis* (*On Triangles of Every Kind*). In it, he writes of plane triangles, proves his Law of Sines, and introduces spherical geometry (Allan). This book however was not published until 54 years after his death in 1530 (Allan). In 1475, Regiomontanus was asked by the Pope to advise on the calendar reform but died before he could take his appointed position [1].

Proof of the Law of Sines:



In triangle ABC, draw CD perpendicular to AB. Then CD is the height *h* of the triangle. The height now separates triangle ABC into two right triangles, CDA and CDB.

(This was necessary, because the trigonometric functions are defined in terms of a *right* triangle.)

We will now show that

$$\frac{\sin A}{\sin B} = \frac{a}{b}.$$

Now, in triangle CDA,

$$\sin A = \frac{h}{b}.$$

While in triangle CDB,

$$\sin B = \frac{h}{a}.$$

Therefore,

$$\frac{\sin A}{\sin B} = \frac{h/b}{h/a} = \frac{h}{b} \cdot \frac{a}{h} = \frac{a}{b}.$$

This is what we wanted to prove.

In the same way, we could prove that

$$\frac{\sin B}{\sin C} = \frac{b}{c}$$

and so on, for any pair of angles and their opposite sides. [10]

VI. Nicolas Chuquet

Nicolas Chuquet (1445 -1500) wrote *Triparty en la science des nombres* in 1484 which was an algebraic book in three parts [1]. This is known to be the earliest French algebra text of which most was already known to the Islamic algebraists. In this work, Chuquet uses and overscoring of the letters p and m for plus and moins (minus) [6] which was later turned into the symbols + and – that we use today by Johann Wildman along with using underlines to group and R^2 and R^3 for square and cube roots [1]. Chuquet also may have been the first to recognize zero and negative numbers as exponents. Also he notes that there are many solutions to systems of three unknowns and two equations [1]. *Triparty* was not printed until 1880.

VII. Luca Pacioli

Luca Pacioli (1445 – 1517) is known as the father of accounting. His work *Summa de arithmetica, geometrica proportioni et proportionalita* (Everything about Arithmetic, Geometry, and Proportions) is the first known accounting textbook. It is also a summary of all known mathematics of the time. Pacioli also was the first to translate and print Euclid's Elements in Latin [1][9].

VIII. Robert Recorde

Robert Recorde (1510 – 1558) was an Englishman who is the first to use two parallel lines as an "equals" symbol. Prior to this a single line or 'ae', short for aequalis meaning equal, was used to denote equals. "Bicause noe 2 thynges can be more equalle" [1]. He wrote many textbooks of which all but one were written in the form of dialog between master and student. The one that wasn't is considered to be an abridged version of Euclid's Elements. Recorde died in prison, sentenced for debt.

IX. Solution of the Cubic

Since around the time of Christ, mathematicians had been working towards a solution of a general cubic equation. Fifteen hundred years later Scipio del Ferro discovered a solution algebraically far truncating the three hundred and some odd years it took to prove Fermat's last theorem. Ferro, however, is not widely known for the discovery of the solution. This is because Ferro could only solve one type of cubic equation since he did not use any negative numbers. Nicolo Tartaglia found a solution to all types of cubics. In fact, he did so after being challenged by Ferro. It was common in this time for scholars to challenge each other in order to gain or maintain positions at their institutions since tenure was unheard of and teachers came and went after only a few years [5]. Tartaglia kept his solution a secret for many years. This was also common practice of this time, keeping discoveries secret meant that you had an advantage if ever challenged. The way the challenges worked is each of the two men would challenge the other with a set number of problems and after a period of time the one who solved the most of the other's problems was declared the winner.

Tartaglia only told his secret to one person, Girolamo Cardano, with the promise that it would never be published. Cardano did however publish it in one of his 230 books, *Ars magna* (Great Art) along with a solution to the quartic by radicals which was also discovered by Tartaglia.

Taraglia's methods were challenged six times by Ludovico Ferrari and finally the two met on August 10, 1548 for a challenge. Ferrari was declared the winner [1].

A solution to the cubic is as follows [1] :

First we transform $ax^3 + bx^2 + cx = d$ into $x^3 + px = q$, first by dividing by a and then using transformations $x = y + \beta$ and then $y = x$.

Next define u and v by

$u - v = x$ and $uv = \frac{1}{3}p$

This gives

$$(u - v)^3 + p(u - v) = q$$

$$u^3 - 3u^3 v + 3uv^3 - v^3 + p(u - v) = q$$

$$u^3 - v^3 - 3uv(u - v) + p(u - v) = q$$

$$u^3 - v^3 = q$$

Using $v = \frac{p}{3}u$, substitute to get :

$$u^3 - \frac{\left(\frac{p}{3}\right)^3}{u^3} = q$$

$$u^6 - qu^3 = \left(\frac{p}{3}\right)^3$$

$$(u^3)^2 - q(u^3) = \left(\frac{p}{3}\right)^3$$

Solving for u:

$$u = \left[\frac{q \pm \sqrt{q^2 + 4\left(\frac{p}{3}\right)^3}}{2} \right]^{1/3}$$

And then for v; finally for x. Now compute $v = \frac{p}{3}u$ and then compute $x = u - v$.

$$x = \sqrt[3]{\sqrt{\left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2} + \frac{q}{2}} - \sqrt[3]{\sqrt{\left(\frac{p}{3}\right)^2 + \left(\frac{q}{2}\right)^2} - \frac{q}{2}}$$

X. Conclusion

Mathematics in the renaissance had a slow start but overcame the hardships of the time to lay the way for many great advancements to come with the help of a few great minds. Shortly after the renaissance, two mathematicians by the names of Newton and Leibnitz used the mathematics of the time to discover calculus, which then led Euler to discover analysis.

Works Cited

[1]Allen, G. D. Dept of Mathematics, Texas A&M University. 11 Mar. 2009 <http://www.math.tamu.edu/~dallen/masters/renaissance/renainsc.pdf>.

[2]Boyer, Carl B. History of mathematics. New York: John Wiley & Sons, 1991.

[3]"Evolution of Renaissance Mathematics:." Www.personal.psu.edu. 11 Mar. 2009 <http://www.personal.psu.edu/jlk465/Evolution%20of%20Renaissance%20Mathematics.htm>.

[4]Field, Judith Veronica. The Invention of Infinity Mathematics and Art in the Renaissance. Oxford: Oxford UP, 1997.

[5]Katz, Victor J. History of mathematics an introduction. New York: HarperCollins, 1993.

[6]Otero, Daniel E. "The Birth of Algebra." Xavier University Computer Science. 12 Mar. 2009 <http://cerebro.xu.edu/math/math147/02f/algebra/algebra.html>.

[7]Radford, Luis. "On the Epistemological Limits of Language: Mathematical Knowledge and Social Practice during the Renaissance." JSTOR: An Error Occurred Setting Your User Cookie. 11 Mar. 2009 <http://www.jstor.org/pss/3483173>.

[8]"Renaissance - Definition from the Merriam-Webster Online Dictionary." Dictionary and Thesaurus - Merriam-Webster Online. 11 Mar. 2009 <http://www.merriam-webster.com/dictionary/renaissance>.

[9]Smith, L. M. "Luca Pacioli: The Father of Accounting." 12 Mar. 2009 <http://acct.tamu.edu/smith/ethics/pacioli.htm>.

[10] Spector, Lawrence "The Law of Sines" 29 Apr. 2009 <http://www.themathpage.com/aTrig/law-of-sines.htm#proof>