Jacob Steiner and Julius Plücker

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Abstract

The foundation of early synthetic and analytical geometry was the start two great minds going in the same direction but on two totally different paths. Jacob Steiner went the path of synthetic geometry and developed many foundations of geometry. Julius Plücker was the analytical geometer which took his talent not only to geometry but also to the study of physics.

1 Introduction

The nineteenth century was full of great things in geometry but could have been so much more rewarding had two men of very similar backgrounds come together as one unit. Jacob Steiner was a bright man who didn't start his education until he was 18 and turned out some of the best synthetic geometry ever seen, but it was also the synthetic geometry that may have kept him away from his greatest discovery Julius Plüker. Julius started his schooling early and went with the analytical approach to geometry and he also had some of the greatest discoveries in geometry but never got to discover Jacob Steiner's work.

2 Jacob Steiner

2.1 Early Life, Education, Teaching and Recognition

On March 18, 1796, Jacob Steiner was born in the small village of Utzensdorf. Utzensdorf which was located in Canton Born, Switzerland, where Steiner's family farmed. Jacob was the baby of the family, having three older brothers and one older sister. Everyone in the family was needed to run the farm and school was second in priority. As a boy, Steiner had no opportunities for education and did not learn to write until the age of fourteen.

Jacob was recognized as a very intellectual kid but it was not until he was eighteen years old that his parents allowed him to go off to school. Jacob traveled to Canton Vaud to study under Pestalozzi in the village of Yverdun. Pestalozzi was a Swiss educational reformer who taught mathematics at his school in Yverdun. Steiner made tremendous progress in mathematics and after only a year and a half was certified to teach mathematics at the school. Steiner taught for a short time and in the fall of 1818 he left for Heidelberg, Germany. There Steiner would begin to give private lessons for a means of support for research and living. A teaching job took him to Berlin in 1821 where he taught for only a year. Next, Steiner realized he could do better with private instruction. Steiner was giving instruction to the sons of Prince August and William Von Humboldt, and was regarded as a great instructor in the city. Next, Steiner was appointed to the prestigious Berlin Gewerbschule, an industrial school in 1827. At, Gewerbschule, Steiner would be a teacher of mathematics and a Royal Prussian professor. This was a high regarded position, which was believed given because of the support of Prince August, William Von Humboldt, and the other prominent families who Steiner gave lessons to their children. Steiner had become friends with A.L. Crelle and Norwegian mathematician Abel while in Berlin.

Abel was regarded as a genius in mathematics while Crelle founded, *Journal fur reine und angewandte Mathematik*, a journal publication created in 1826 that still exists today. The first volume of the periodical both Abel and Steiner had contributions, Steiner had four papers published. This opened the gates for more and more publications and new discoveries. Steiner started his first book when he was 26 years old and worked on it for four years. The book was about spheres and circles and the theory of contact and intersection. The time that Steiner was writing this book a student at the University of Berlin by the name of C.G.J. Jacobi would become one of his students who would make significant contributions to geometry research. Steiner is said to have laid the foundation of modern synthetic geometry in 1832 in his first volume published. This volume was so great that Steiner would be given honorary degree of doctor of Philosophy from the University of Konigsberg in 1833. Then the University of Berlin would make him a chair of geometry, and he would be elected a fellow of the Prussian Academy of Sciences. The third work to be published was, *Geometrical Constructions with a Ruler*, at the age of 38 and shortly before becoming University of Berlin chair in geometry. Steiner's papers were for the most part results of what he had discovered or found. Steiner dealt only with synthetic geometry and would not dare do anything analytically. The lectures that Steiner presented in Berlin of synthetic geometry were published after his death and were believed to complete the volume he started earlier in his life. Some believe Steiner's most important paper was on general properties of algebraic curves which related to curves and surfaces. this paper however has fascinating results which is the problem that it only contained results. Some of the theorems were proved using geometry analysts who had strong resiliency to find their own way. A geometer by the name L. Cremona proved all of them, using a synthetic method very similar and uniformed. K. Weierstrass obtained and edited Steiner's papers, then the Prussian Academy of Sciences published the edited works in 1881.

Jacob Steiner became ill and spent the latter part of his life in Switzerland but always holding the position of chair of geometry at the University of Berlin. Jacob Steiner died in Bern, Switzerland on April 1, 1863. Steiner's personal papers were given to the Bern library, which contained scientific and personal information. These papers were discovered 30 years later and published in 1929 as, *Unpublished Steiner manuscripts* by Professor J.H. Graf. Steiner left about one-third of his money to the Prussian Academy of Sciences which was used to form a Steiner Prize for Geometry.

Figure 1: Jacob Steiner

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3 Julius Plücker

3.1 Early Life, Education, Teaching and Recognition

Julius Plücker was born June 16, 1801 in Elberfeld, Duchy of Berg which is now Wuppertal, Germany. Julius grew up in Dusseldorf and attended the gymnasium or local school there. Julius's family who came from Aachen and were merchants gave him French and German bloodlines. This is clear because it shows how he prefers how Monge the French mathematician does geometry. After the gymnasium Julius then finds the way of his German heritage by attending numerous universities as was common for German young men.

The first stop for Julius was the University of Bonn, then Heidelberg and then the University of Berlin. In 1823 he traveled to France and the University of Paris for courses on geometry. In 1823 Julius submitted his doctoral dissertation to the University of Marburg. Then in 1824 he submitted his thesis for habilitation to the University of Bonn and was given designation as a dozent. Four years later the University of Bonn gave Julius title of extraordinary professor. Then in 1833 after five years as extraordinary professor Julius left for Berlin and Julius was extraordinary professor at the University of Berlin and was a teacher at Friedrich Wilhelm Gymnasium for a year. An extraordinary professor at the University of Berlin at the time was a dream job for any mathematician, except Julius. The chair of geometry had just been given to a young man named Jacob Steiner who had strong personality and a brilliance for synthetic geometry. Julius thought more analytically and respected synthetic geometry but wanted to study analytical geometry. The two men could find no common ground and Julius knew that it could only get worse for him since Steiner was in better with the high ranking people in the school. Julius started looking to go other places to avoid conflict with Steiner and advance his research. Julius took a step down and backwards as he became an ordinary professor at Halle in 1834. Then after two years Julius went back to Rheinische Friedrich-Wilhelms University of Bonn and took the position of mathematics chair.

Julius life had stared to settle down after he left Berlin and in 1837 he married Miss.Altstatten. Julius started research in the applications of geometry and would become a father as they would have a son. Julius passion was geometry but to him geometry needed to be applied and used in real world situations. Then after studying geometry all his life, Julius made a drastic change in 1847 to physics. Julius took the chair of physics and although he was not a great experimenter, he could teach physics to a high degree of success. As the chair of physics Julius worked on atomic physics a great deal. Julius published manuscripts in magnetism and atomic physics. Like Steiner, Julius never gave up the chair of physics as he wouldn't relinquish it until his death. Although he held the chair of physics, Julius did not leave his passion of geometry. Through the 18 ears as chair, Julius still produced major analytical work in geometry. Julius major contributions include conic sections with point and line coordinates applied to them, what happens to algebraic curves at infinite points, and Plücker equations which tells class and order of a curve. Those are just some of the early contributions made by Julius.

In 1866, Julius getting older took on an assistant named Klein who would be with him till the end. Julius Plücker died May 22, 1868 in Bonn, Germany. Julius had just won the Copley Medal in 1866 and just published a new work just months before his death.

Figure 2: Julius Plücker

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4 Steiner's Works

4.1 Synthetic Geometry

Ernest P. Lane said, "Synthetic geometry is pure geometry,[8]." To understand Jacob Steiner, one must first understand synthetic geometry. Synthetic geometry uses compasses, straightedges, and practicality. There are is no analysis in synthetic geometry, there is only logic and reason. To get logic and reason though a person must be able to think intuitively or outside the box. Synthetic geometry makes the world round because it gives an answer to everything concrete or real. Jeremy Gray said of Poncelet a great geometer, "He regretted that the arguments used in synthetic geometry lacked generality,[1]." So, Poncelet, a great geometer used synthetic geometry but knew that it was for geometry that could be measured and recorded.

This was the kind of man Jacob Steiner was, a man of reason, logic, being practical, and using geometry for things that could be solved. Though Poncelet regrets the lack of generality, it takes nothing away from the brilliancy of Jacob Steiner. Synthetic geometry now means something different that what it did when Steiner and Poncelet used it. Synthetic geometry back then meant that algebra was left out and curves, lines, and angles would be presented and discussed. Jacob Steiner only practiced, taught, and researched in synthetic geometry. This shows that he thought little of any of analytical geometry, and gives an insightful look into his personality. Steiner's personality was stubborn, serious, and not open to suggestion in geometry that did not use synthetic geometry.

4.2 Steiner Terms

Jacob Steiner has contributed many terms to geometry because of his brilliant research and capacity to think logically. First, are Steiner points which are useful for may things, hence why Steiner points are referred to first. A Steiner point is defined in Steiner's book as " the fifteen points in which the 60 Pascal lines of a hexagon 123456 inscribed in a conic meet by threes,[2]." To give an example of a Steiner point look at 123456, 143652, 163254 and a Steiner point is given. Steiner points are mergers of three lines forming three 120 degree angles and has been proven for all Steiner points. This is helpful when talking about Steiner tree's. Minimum Steiner tree's are used and the definition states they are "the shortest network, allowing Steiner points,[4]." A Steiner ellipse, which taking a triangle and the smallest ellipse to circumscribe the triangle. A Steiner circle is defined as "the circle through any point P of a conic and also the tree points where tree other circles through P osculate the conic elsewhere,[2]." A Steinerian, defined as " the locus of a point whose point whose first polar curve with respect to a given algebraic curve has a double point,[2]."

4.3 Steiner's Roman Surface

Steiner's Roman surface is defined as " of fourth degree and third class, and such that every tangent plane to it intersects the surface in two conics,[2]." Great mathematicians have studied Steiner's Roman surface for research and other findings but Steiner was the first to come across them. In 1844, jacob Steiner traveled to Rome to visit and was looking for the Steiner Roman surface which at the time did not exist but he found it in Rome. It was given the Roman because whenever he found it he was in Rome. Steiner was looking for a surface whose plane sections all were conics and not of degree 2 or a quadric surface. The surface had to be made up of conics and Steiner looked at surfaces who had degree four. One equation for it is:

$$(x^2 + y^2 + z^2 + w^2 - 2xy - 2xz - 2xw - 2yz - 2yw - 2zw)^2 = 64xyzw$$

The Steiner Roman surface was the foundation that helped a mathematician by the name of Veronese. Veronese was a colleague of Klein who was Julius Plücker's assistant in the last three years of his life. Veronese used Steiner's Roman surface to help him solve for the Veronese surface.

5 Plücker's Works

5.1 Analytical Geometry

In analytical geometry to demonstrate geometrical theorems one would use analysis and algebras with the introduction of a coordinate system. This is very different from synthetic because analytical geometry deals with lines of infinity, infinite space, analysis, algebra, and abstract mathematics. Julius worked with analytical geometry almost exclusively for both his research in geometry and in physics. Julius theorems were presented in an elegant analytical geometry that was demonstrated with algebra and analysis. The algebraic symbols used in conic section equations and the smooth way his operations were done show his passion for the analytical geometry. Julius had a strong desire for analytical geometry and it is shown in his brilliant work. The analytical side molded Julius into one of the most insightful geometers the world has ever seen. Julius used analytical geometry for physics because one of his passions was to apply geometry to physical sciences.

5.2 Julius Publishing's

Julius had a wide range of talent in geometry. This is evident in the works that he is responsible for writing and having published. The first thing Julius published was , "Analytisch-geometrische Enivickelungen," which would have to be published in two volumes in 1828 and 1831. These volumes were of plane analytical geometry, which covered conic sections, circles, and lines. Then in 1835 and 1839 Julius published two more major works both using the fluid analytical geometry as the first two had. These works would cover infinite points and the properties of algebraic curves near those points, singular points on the plane, conic sections and applying point and line coordinates on conic sections, and class and order of a curve also known as 'Plücker equations'. Then in 1857, *Magnetic Induction of Crystals*, was published and was not one of his usual works. This journal still used the abstract format that Julius used but was much more on the physics side other than that of geometry. Then in 1865 a journal was published called, "On New Geometry of Space", and Julius was back to analytical geometry of infinite space. This was a complex work put together using analysis and algebra to talk about infinite space and linear configuration. Then in 1866 Julius produced, "Fundamental Views regarding Mechanics", which was a 20 page journal of force represented geometrically. In, "Fundamental Views regarding Mechanics," Julius uses algebra and analysis to present forces acting on a linear complex, on congruency of forces, and the way force acts on a plane. Julius publishing's have offered much insight into the world of geometry and physics. These works have helped other great geometers like Klein, Salmon, and Hilbert.

6 Conclusion

Jacob Steiner and Julius Plücker were two of the most gifted minds in the history of geometry. The problem was that they conflicting personalities and styles. The world can only wonder how things could have been different had these two put there differences aside and joined forces. After looking at the major contributions of both and the contrasting styles, the world of mathematics would be even more enlightening had the two worked together. The two couldn't come together and although we may wonder what could have been, they were still able to prove great things in geometry and physics.

References

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