Early and Egyptian Mathematics

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April 30, 2009

Abstract

The earliest mathematics came from necessity. Mathematics did not truly start to advance until the Egyptians. The Egyptians did a lot to help kick start the advancement of mathematics. They were able to take mathematics that they developed and apply it to help create some of the most spectacular structures.

1 Introduction

There is a lot of early mathematics that is hard to figure out. Most information that has been found from thousands of years ago is either missing or incomplete. Most of the early information that has been discovered has come from the Egyptians. The Egyptians are most well known for the pyramids and controlling the flooding from the Nile river. These achievements where done by the development and advancements in mathematics.

2 The Origin of Mathematics

Mathematics came by way of necessity and leisure time. People of the era needed a way to keep track of objects, exchange goods fairly, and to create new technologies. For example, herders needed to know how many animals were in their herd at the beginning of their trip, and then again at the end of their trip. This is what started the evolution of counting and number bases.

Table 1: Most Common Number Bases

Name of Base	Base Number	Digits
Decimal	Base 10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Quinary	Base 5	0, 1, 2, 3, 4
Binary	Base 2	0, 1
Tenary	Base 3	0, 1, 2
Vigesimal	Base 20	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F, G, H, I, J

2.1 Counting

Counting originated by assocaiting a one to one coorispondence, such as one animal for one stick, two animals for two sticks and so on. However, as people needed to keep track of larger groups of objects, they would cut notches onto animal bones or pieces of wood for each object in order to keep track of them.

These bones and pieces of wood are some of the oldest artifacts that have been found and dated. One of oldest mathematical artifacts was discovered in the early 1970's on a site of a Border Cave in the Lebombo Mountains between South Africa and Swaziland. It was a piece of baboon fibula, that has been dated to 35000 B.C. [3].

"Darwin suggest in the Descent of Man (1871) that certain animals can distinguish between number, size, order and form" [10]. Which do to modern experiments, done with animals and insects, people have been able to see a picture of how humans were able to start counting. Crows were found to be able to distinguish sets containing up to four objects, and Monkeys were found to be able to count to at least nine.

2.2 Bases

After basic counting was established people had different ways of counting. This evolved into the number base. There are many different number bases that have developed throughout this era. The most common base that is used today is base ten, or decimal. The other common bases are found in Table 1. Bases have the number of digits n from $0, 1, 2, 3, ..., n$ such that n is the base number.

It is possible to convert between bases as well. The easiest way to do this is by converting to base 10 in between the different bases. This is because we use base 10 the most, therefore we understand how to work with it the best.

An example of converting $(1346)_7$ to base 10:

$$\begin{gathered}6(7^0) + 4(7^1) + 3(7^2) + 1(7^3) \\ = 6(1) + 4(7) + 3(49) + 1(343) \\ = 6 + 28 + 147 + 343 \\ = 524\end{gathered}$$

So $(1346)_7 = (524)_{10}$.

Then an example of converting the other way from $(524)_{10}$ to base 7. This is done by filling out the following table.

$7^3 = 343$	$7^2 = 49$	$7^1 = 7$	$7^0 = 1$

First step:
$524 \div 343 = 1$ R 181

1			
$7^3 = 343$	$7^2 = 49$	$7^1 = 7$	$7^0 = 1$

Second step:
$181 \div 49 = 3$ R 34

1	3		
343	49	7	1

Third step:
$34 \div 7 = 4$ R 6

1	3	4	
343	49	7	1

Fourth step:
$6 \div 1 = 6$ R 0

1	3	4	6
343	49	7	1

So $(524)_{10} = (1346)_7$.

2.3 Writings

The mathematics and other writings at the time where done in two main forms which were cuniform tablets and on papyrus scrolls. Some of the oldest records are from Egypt, and their records have survived 4,000 years due to Cuneiform[1] [2]. Most of these that are found are cracked or broken. Papurus scrolls are long sheets of paper like material. The material is usually made out of tree bark or leather. These are usually found torn or missing pieces.

3 Egyptian Mathematics

The Egyptians took control of most of the beginning development of mathematics. What we know from this era is mainly from old papyrus and tabelts that have been found thousands of years later. These scripts were damaged and missing information or we may have. This means that our information may not be as correct as possible.

The Egyptians used a 10-scale base and they carved their hieroglyphics[2] on stone or wood tablets. The Egyptians where also known for their accuracy in counting and measuring, and the pyramids are a great example of that.

3.1 Calendar

The Egyptians observed that the annual flooding of the Nile took place after Sirius[3] rose in the east sky. The Egyptians then realized this event happened every 365 days. So they created a calendar of 12 months of 30 days

[1] Cuneiform is where wedge-shaped marks are carved on soft clay tablets that are baked in ovens or by baking in the heat of the sun.
[2] Hieroglyphics can be written vertically, left to right, or right to left.
[3] The Dogstar

each and 5 extra feasting days in order to deal with the flooding. They also realized that the civil year was short by a quarter of a day. So to compensate for this they advanced their calendar by one day every 4 years, leap year. This then allowed the seasons coincide with the days on the calendar[2] .

3.2 Geometry

"Herodotus believes that geometry started in Egypt due to the practical needs of surveying after annual flooding[4] " [2] . "Aristotle believes that geometry started in Egypt due to the existance of the priestly leisure class" [2]. However, both could be wrong about how geometry developed in Egypt. Ancient pottery, weaving, and baskets show the use of geometry earlier than the Egpytions. They seemed to be familiar with the Pythagorean Theorem, but the did not actually create it. Instead, they seemed to work with ratios of the sides of the triangle.

They also created a formula for the area of a triangle:

$$Area = \left(\frac{1}{2}\right) bh$$

3.3 Area of a Circle

The Egyptians used the area of square to find an approximation for the are of a circle. They did this by a simular method to exhaustion[5]. They placed an octagon inside a square, and they found the area of the square. Then they subtracted the 4 triangles on the corners. This gave them their approximate area. After using this method they where able to get an assumption for π, their assumption of π is very close the the real assumption. Their assumtion was:

$$\pi = 3\frac{1}{6}$$

[4]These geometer's where known as rope strechers or surveyers.

[5]Exhaustion is a method by which an operation is repeatedly done till there is an accurate enough approximation.

The Egyptians ability to have such a close approximation o π is further proof of how accurate the Egyptians werewth their measuring.

4 The Rhind Mathematical Papyrus

The Rhind Papyrus was found in Thebes near Ramesseum. It was later purchased in 1858 by A. Henry Rhind. He saw the importance of the piece and after his death he donated it to the British Museum. They named the papyrus after him [4].

It was found later that the Papyrus was most likely a copy that was scribed by A'hmosè. The date on the Papyrus was indicated in the method that the ancient Egyptians used. This method was where the year was noted by the Pharaoh's name. The date on the Papyrus was indicated under the Pharaoh A-user-Rê. He was apart of the Hyksos dynasty which was dated around 1650 B.C. [4].

4.1 Problem 7

As an example of the problems on this papyrus, I picked problem number 7 of the Rhind Mathematical Papyrus which comes from the 3^{rd} section which is the section titled "Multiplication by Certain Fractional Expressions."

Problem 7:
As it appears in the papyrus translated to english: [4]

$$\textit{Multiply } \frac{1}{4}\ \frac{1}{28}\ \textit{by } 1\ \ \frac{1}{2}\ \ \frac{1}{4}.$$

$$\begin{array}{ll} 1 & \frac{1}{4}\ \ \frac{1}{28} \\ \frac{1}{2} & \frac{1}{8}\ \ \frac{1}{56} \\ \frac{1}{4} & \frac{1}{16}\ \ \frac{1}{112}\ \textit{as parts of } 28 \textit{ these are } 1\ \ \frac{1}{2}\ \ \frac{1}{4} \textit{ and } \frac{1}{4} \\ \textit{Total} & \frac{1}{2} \end{array}$$

5 Unit Fractions

Unit factions are that have unit numerals in the numberator. The Egyptians only used unit fractions in their arithmetic. The only non unit fraction that was used is $\frac{2}{3}$. The Egyptians also made several tabels to help them with their arithmatic. They had several tables for unit fraction calculations. For example the $\frac{2}{n}$ table. This table is thought to be made by one of the two following formulas:

$$\frac{2}{n} = \frac{1}{\frac{n+1}{2}} + \frac{1}{\frac{n(n+1)}{2}}$$

or

$$\frac{2}{p \cdot q} = \frac{1}{p \cdot \frac{p+q}{2}} + \frac{1}{q \cdot \frac{p+q}{2}}$$

6 Weakness

There are many debates as to if the Egytians are as great of mathematicians as everyone says. However, there are those that believe that because of the pyramids, the Egyptians were seen as mathematic's elite when they really where not.

The Egyptians had many weaknesses, all of their mathematics is built only around addition and most problems where only practical real life situations. Their calculations are the main elements in their problems. Their theoretical elements dealt with technique rather than understanding what the problem is asking.

Their rules for calculation are for specific concrete cases only, and the main sources of information from the time may have been only intended for students or made by students. So our sources may not be as accurate as we would hope for what is really known by the Egyptians during this time. There was not much change in the Egyptian's mathematics throughout its long history.

7 Conclusion

The Egyptians had many achivements. The pyramids are proof of their advancement in triangles development and their incredible accuracy. Even though there are sceptics as to wither or not the Egyptians are overly praised for their achivments, without them mathematics would not be where it is today.

References

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