

Kurt Gödel and His Theorems

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Abstract

Kurt Gödel is a man who escaped from the Nazis to come to America where he would strive in his findings in mathematics. However, like many other great mathematicians he would find himself losing his mind and living a life as a hyperchondriac. When looking back on his accomplishments he is best known for his incompleteness theorems, but that is not all that he accomplished in his lifetime.

1 Introduction

“He is the most famous mathematician that you have most likely never heard of” (Goldstein 22). At the age of 23, Kurt Gödel made two incompleteness theorems that are logically related, and he disproved one of the most talked about mathematicians, David Hilbert. This was only the beginning of what was to come. “The greatest mathematical logician of all time and certainly the maddest” (Goldstein). He lived a life as a hyperchondriac, which eventually would kill him.

2 Background

Kurt Gödel was born on April 28, 1906 in Brünn, Moravia to Rudolf and Marianne Gödel. His protestant ethnic German family encouraged him to increase his knowledge for math and science because no one else in his family

seemed to have an aptitude for them.

Rudolf Gödel was asked to look back on the time that he spent with his little brother, Kurt.

”Although my brother was perhaps less close to the family and went his own way earlier, he was also—particularly later on when he had become rather sickly—mother’s special problem-child.”

—Rudolf Gödel (Casti, 58)

Rudolf believes that he knows the source of Gödel’s hypochondria. He believes that it is a result of a case of rheumatic fever that Gödel went through when he was eight years old. After he recovered Rudolf remembers Gödel researching the long-term affects of the disease. He believed that his heart had been affected by the disease (Casti). So he began to over think any type of injury or illness he recieved.

Gödel finished his first eight years of school in only four years, so he attended Realgymnasium, which is equivalent to high school, in 1918 when he was 12 years old. He was a good student and recieved very high marks in all of his classes, except for one of his math classes.

After high school in the Fall of 1924 he decided to attend the University of Vienna with his older brother, Rudolf Gödel. Kurt Gödel planned on pursuing a career in Physics while he was attending the University of Vienna. However, he strayed away from physics to mathematics because of his interest in precision. His interest in mathematics later narrowed onto mathematical logic because of his sparked interest in number theory.

3 Disproving Hilbert

Gödel is credited with discovering some of the most controversial theorems, and by disproving the mathematical communities “norms”.

One of the most controversial was that the methods of deductive logic are not strong enough for all facts: “The truth is simply bigger than the proof,”

(Casti). He then presented his findings at a conference in Königsberg on “Epistemology of the Exact Sciences” which was organized by *Gesellschaft für empirische Philosophi*, or in English the Organization for Empirical Philosophy. He also published his findings in an unusual manner:

Gödel’s Theorem - Informal Version

Arithmetic is not completely formalizable (Casti)

Later he decided to publish a formal version of the above theorem:

Gödel’s Theorem - Formal Logic Version

For every consistent formalization of arithmetic, there exist arithmetic truths that are not provable within that formal system. (Casti)

This basically meant that there are no rules for generating all of the truths about the natural numbers. Which went against everything that David Hilbert had been working on. People then began to question Hilbert and what he was telling them.

4 Incompleteness Theorems

Gödel started his doctoral dissertation in early 1929. He decided to complete his dissertation on the completeness of limpid logic. His father passed away unexpectedly while he was working on his dissertation. This just gave him more motivation to keep his mind on his work. He discovered two incompleteness theorems that seemed to be extremely complicated, but the overall strategy was not. It was surprisingly simple.

“Gödel’s theorems, then, appear to be that rarest of rare creatures: mathematical truths that also address themselves—however ambiguously and controversially—to the central question of the humanities: what is involved in our being human?” (Goldstein 26).

4.1 First Incompleteness Theorem

At the same conference where Gödel disproved Hilbert he had something else that was ready to be presented. He saved this new theorem for the last day of the conference. This was the theorem that made him famous. He only had one sentence to present.

“One can even give examples of propositions which are really contextually true, but unprovable in the formal system of classical mathematics”

–Kurt Gödel (Goldstein 23)

The proof of this sentence is about twenty pages long. Within the proof he comes close to self contradiction because “he is proving that there are true arithmetical propositions that are not provable” (Goldstein). The strategy for the proof comes from the liar’s paradox (“This sentence is false.”). To avoid the paradox he worked on the provability of the statement rather than if the statement was true.

4.2 Second Incompleteness Theorem

Gödel’s second incompleteness theorem came shortly after his first. The *Encyclopedia of Philosophy’s* article “Gödel’s Theorem” states:

A corollary to the theorem is that the consistency of a formal system adequate for number theory cannot be proved within the system (Goldstein 23).

Basically the theorem states, “no sufficiently strong consistent mathematical theory can prove its own consistency” (Jech).

Table 1: Gödel numbering of elementary logical signs (Casti)

Sign	Gödel Number	Meaning
\sim	1	not
\vee	2	or
\rightarrow	3	if...then
\exists	4	there exists
$=$	5	equals
0	6	zero
s	7	the immediate successor of
(8	punctuation
)	9	punctuation
'	10	prime

5 Gödel's Notation

Gödel did not use number or formulas to prove his theorems, instead he was one of the first mathematicians to use almost entirely mathematical symbols for his proof. He used formulas of number theory for the representation of real numbers. Tabel 1 is an example of some of Gödel's most used symbols.

6 Gödel Comes to the United States

On March 13, 1938 Austria annexed to Germany. Gödel saw this as his chance to share his knowledge with others. He became a guest lecturer at Notre Dame University. He started to go back and forth between the United States and Vienna, where his wife was still living. However, Gödel was no longer comfortable with going back to Vienna, due to the Nazi's rising to power. So Gödel and his wife left for the United States. They had to travel across Russia and Japan in order to avoid the Nazis, and they ended up in San Francisco. When he arrived in the United States he began to teach at Princeton University. Later on he also lectured at Brown University and Yale University. He gave lectures on many of his favorite topics, such as set theory.

Gödel and his wife became United States Citizens on April 2, 1948. There is a story that Gödel found an error in the United States Constitution. When Gödel found the mistake he immediately told his wife and Einstein about it. Gödel thought that he should make it aware to the people that were going to swear him in, but Gödel's wife and Einstein told him not to do so because they were afraid that they might not give him his citizenship. When he was asked about the United States Consitution he felt that it was his duty to tell them what the problem was. The people then became very concerned and took another look at Gödel, but in the end Gödel was still granted citizenship.

7 Gödel and Einstein

Gödel met Albert Einstein around 1942. They had many things in common, they both fled Germany in order to escape the Nazi's power, and they were also both very interested in each other's work. They eventually became very close friends till Einstein's death in 1955. Before Einstein's death they would take long walks together to talk about their theories and theorems they were working on.

8 End of Gödel

During the last few years of Gödel's life he became sick. He was a hypochondriac for most of his life, but he delayed the proper treatment of a bleeding ulcer in 1945. This forced him to have a blood transfusion. This event lead him to be skeptical of people and eventually drove him mad. He believed that someone was trying to kill him by poisoning his food. He would then only eat food if his wife tasted it first. When his wife died he had no one that he would trust to taste his food for him. So he died in the hospital on January 14, 1978 of malnutrition.

9 Conclusion

Gödel is one of the greatest mathematicians of the 20th century. He was able to turn the mathematical world upside down. His thinking was like no other mathematician of his time. Even though he spent his life as a hyperchondriac afraid that someone was out to get him, he was still able to impress the minds of some of the greatest thinkers of all time. He seemed to live by the motto:

“But every error is due to extraneous factors (such as emotion and education); reason itself does not err.”

–Kurt Gödel

29 November 1972

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