President Garfield and his Pythagorean Theorem Proof

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Abstract

This paper will give personal and educational background of President Garfield's life. In addition to discussing the Pythagorean theorem, this paper will include proofs of the Pythagorean Theorem, including the President's Proof.

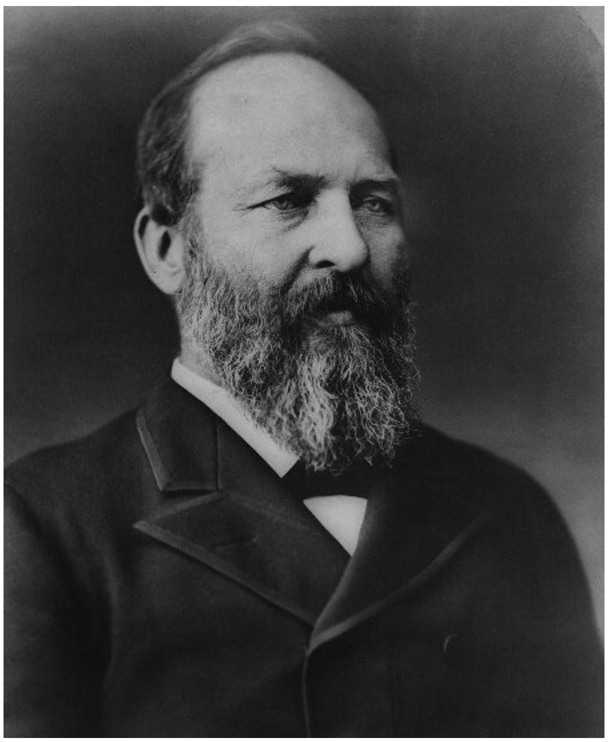
1.jpg 

Figure 1: President Garfield

Commend me to the friend that comes
When I am sad and lone,
And Makes the anguish of my heart
The suffering of his own;
Who coldly shuns the glittering throng
At pleasure's gay levee,
And give his heart to me.

He hears me count my sorrows o'er;
And when the task is done

He freely gives me all I ask,-
A sigh for every one.
He cannot wear a smiling face
When mine is touched with gloom,
But like the violet seeks to cheer
The midnight with perfume.

Commend me to that generous heart
Which like the pine on high
Uplifts the same unvarying brow
To every change of sky;
Whose friendship does not fade away
When wintry tempest blow,
But like the winter's icy crown
Looks greener through the snow.

He flies not with the flitting stork,
That seeks a southern sky,
But lingers where the wounded bird
Hath laid him down to die.
Oh, such a friend! He is in truth,
What e'er his lot may be,
A rainbow on the storm of life,
An anchor on its sea

President Garfield's favorite Verses -unknown author [7]

1 Introduction

James Abram Garfield (1831-1881) is considered the last President to be born in a log cabin. Garfield was born from uneducated parents and went on to be a scholar, a Civil was hero, a U.S. congressman, and president of the United States. This 20th President of the United States of America only served as president for a little more than six months. His rags-to-riches life came to a halt when he was shot four months after taking the office of the Presidency. He made contributions to Education and published one of the most-taught proofs of one of the most important theorems in geometry. The Pythagorean Theorem is undoubtedly very important in geometry. Even though the practical applications were known long before the time of Pythagoras, he generalized it. There are infinite many applications and proofs of the Pythagorean theorem. I will show a few applications and proofs, including President Garfield's proof.

2 Personal Background

James Abram Garfield was born in Orange, Ohio on November 19th of 1831. He was the youngest of five children of Eliza and Abram Garfield. Abram Garfield was a farmer and also worked on the railroad to make ends meet. When James was only eighteen months old his father caught a cold and died. Eliza endured tough times raising her children and life was difficult for the Garfield family. All of the children had to work at an early age while they attended school. At age 16, Garfield left home looking for a better job. In Cleveland he was hired to work on a canal boat where he carried cargo between Cleveland and Pittsburgh.[2] His duties included leading the horses that pulled the boat along the canal. His career as a sailor ended when he became ill with malaria.[2] When young Garfield recovered he decided that he wanted to teach. To support himself in his studies, Garfield taught school and also worked as a janitor.[2]

While garfield was teaching in Hiram College, he studied law. He became interested in politics and spoke out about the problems facing the country at the time. The 1850's were years of conflict between the North and South over slavery and the states' rights. Garfield joined the newly formed Republican Party, which had been founded in 1854 in opposition to the expansion of slavery in the western territories of the United States. In 1859, he was elected to the Ohio Senate. There he was able to denounced slavery and called for the preservation of the Union. These were busy years for James Garfield. Besides his office duties, he was a preacher in the Disciples of Christ Church and in 1858 he married Lucretia Rudolph. "Crete," as he called her, had been his childhood friend, a fellow student, and pupil. James and Lucretia had seven children in all, of whom two died as infants.

Garfield volunteered and served with distinction as a soldier in the Civil War from 1861 to 1865, rising to the rank of major general in the Union Army. He received a commission as lieutenant colonel and helped raise a regiment of volunteers. Many of the men were his old students. Garfield had no military experience, but he was willing to learn. He studied military textbooks, and he drilled his men with a textbook in one hand. In December 1861, Garfield was given command of a brigade. He was ordered to attack the Confederate forces under General Humphrey Marshall, an experienced soldier. At the battle of Middle Creek he defeated Marshall and forced him to retreat from Kentucky. It was not a great victory, but it was welcome news in the North, for up to this time the Union Army had won few victories. Garfield was promoted to brigadier general and fought at the battle of Shiloh in Tennessee. While his military reputation was growing Garfield became ill and had to leave the field. But he was soon active again as chief of staff. He fought at Chickamauga in Georgia and for his courage and leadership was promoted to major general.[2]

In 1862, while still in the Army, Garfield was elected to the U.S. House of Representatives. He remained in the Army until December 1863, when he resigned his commission and took his seat in Congress.[2] Garfield served in the House of Representatives for 17 years, including a period as House minority leader. He was particularly interested in matters affecting the freed blacks in the South and in education.[2] In 1880, Garfield was elected to the U.S. Senate. Before he could take his seat, however, he unexpectedly won the Republican presidential nomination[2]. On March 4th of 1881 Garfield was inaugurated as the 20th President of the United States of America. Garfield began his administration as the head of a divided party.

3 Education

I would rather be beaten in Right that succeed in Wrong. James A. Garfield. [1]

James A. Garfield started school at the age of three, attending classes in a log hut. He learned to read and began a habit of reading that would only end when his life ended.[2] By the time he was fourteen, young Garfield was fairly knowledgeable in arithmetic and grammar and was particularly interested in the facts of American history.[2] In fact, he read and reread every book the scanty libraries of his part of the wilderness supplied, and many he learned by heart.[2] In 1849, Garfields mother persuaded him to enter Geauga Academy in Chester, Ohio.[2] In 1851, after finishing his studied in Chester, he entered the principal educational institution of the Campbellites, Hiram Eclectic Institute.[2] He was not a very quick study, but he was determined and he soon had an excellent knowledge of Latin and was fairly adept in algebra, natural philosophy and botany.[2] He read with appreciation, but his superiority was easily recognized in the debating societies of the college, where he was industrious and outstanding.[3] Living at Hiram was inexpensive, and he easily made enough to cover his expenses by teaching in the English department. He also gave instruction in ancient languages.[2] James A. Garfield entered Williams College in the autumn of 1854 and graduated with the highest honors in the class of 1856.[2] On his return to Ohio after his graduation in 1856, he resumed his place as a teacher of Latin and Greek at Hiram, and the next year (1857) being then only twenty-six years of age, he was made its president.[2] He discussed with his interested classes almost every subject of current interest in science, religion, education,art and law.[2]

4 Garfield's death

On the morning of July 2, 1881, Garfield was preparing to leave Washington to visit Williams College. As they waited at the Washington railroad station, a man approached Garfield from behind and shot him twice.[2] The first bullet went through Garfield's arm and the second struck him in the right side of the back.[6] The man, Charles J. Guiteau, was a Stalwart who had been refused a government post. At the time, without the benefit of modern diagnostics, Garfield's doctors could no determine the location of the bullet.[6] At least a dozen medical experts probed the president's wound, often with unsterilized metal instruments or bare hands, as was common at the time.[6] Historians agree that massive infection, which resulted from unsterile practices, contributed to Garfield's death.[6] Guiteau himself repeatedly criticized Garfield's doctors, suggesting that they were the ones who killed the president "I just shot him," Guiteau said.[6] Garfield was nursed at the White House and then at a summer resort cottage at Elberon, New Jersey, where his family was staying.[2] President garfield died on September 19th of 1881 at Elberon and was buried in Cleveland, Ohio.

5 The Pythagorean Theorem

> *In Mathematics the man who is ignorant of what Pythagoras said in Croton in 500 B.C. about the square on the longest side of a right-angled triangle, or who forgets what someone in Czechoslovakia proved last week about inequalities, is likely to be lost. The whole terrific mass of well-established Mathematics, from the ancient Babylonians to the modern Japanese, is as good today as it ever was.* E.T. Bell.[5]

Theorem 1 *If a and b are the lengths of the legs of a right triangle and c is the length of the hypothenuse, then*

$$a^2 + b^2 = c^2$$

This celebrated proposition is one of the most important theorems in the whole realm of geometry and is known in history as the 47th proposition, that being its number in the first book of Euclid's *Elements*.[5] The practical application of this theorem was known long before the time of

Pythagoras. He, doubtless, generalized it from an Egyptian rule of thumb ($3^2 + 4^2 = 5^2$) and first demonstrated it about 540 B.C. from which it is generally known as the Pythagorean Proposition.[5]

Some scholars have argued that the astronomically related stone temples in England built in the third millenium B.C. were constructed using a knowledge of the Pythagorean theorem and, in particular, Pythagorean triples (a, b, c) such that $a^2 + b^2 = c^2$. But the evidence for this is rather tenuous.[4] There is much more substantial evidence of interest in Pythagorean triples, however, in the Babylonian tablet labeled Plimpton 322 (Figure 2), which dates from approximately 1700 B.C.[4] The tablet consists of four columns of numbers that contain in each row two of the three numbers of a Pythagorean triple.[4] The third Pythagorean triple can be found by using the other two columns.[4]

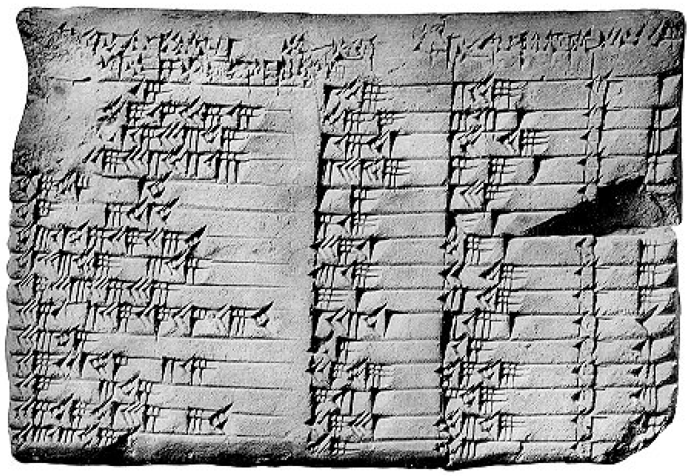


Figure 2: The Plimpton Tablet

5.1 Applications

The Pythagorean theorem is useful in many ways. I will show just a few of the many possible aplications of the Proposition. The most practical application of the Pythagorean theorem is actually an application of its converse. The converse says that:

Theorem 1 *If a triangle has sides of length a, b and c and $a^2 + b^2 = c^2$ then the angle between the sides of length a and b is a right angle.[5]*

Such a triple of numbers is called a Pythagorean triple, for instance $3, 4, 5$ is a Pythagorean triple. The Pythagorean triple provides an easy and expensive way to get a corner square for a foreman in construction. He can drive a stake at the desired corner and another stake 3 meters from the corner along the line where you want one wall of the building. Then position a third stake so that its distance from the corner is 4 meters and the third side of the triangle formed by the three stakes is 5 meters . Since $3, 4,$ and 5 is a Pythagorean triple, the angle at the corner is a right angle.

The Pythagorean Theorem can be used for finding the distance between points in a plane. Consider points $(1, 2)$ and $(4, 6)$ and using the Pythagorean Theorem:

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$D = \sqrt{(4 - 1)^2 + (6 - 2)^2}$$
$$D = 25$$

Another aplication can be found in baseball where the diamond is really a square(figure 3). So we can use the Pythagorean Theorem to answer a multitude of problems like, "How far does a catcher have to throw the ball to get from home plate to 2nd base?" Using the Pythagorean

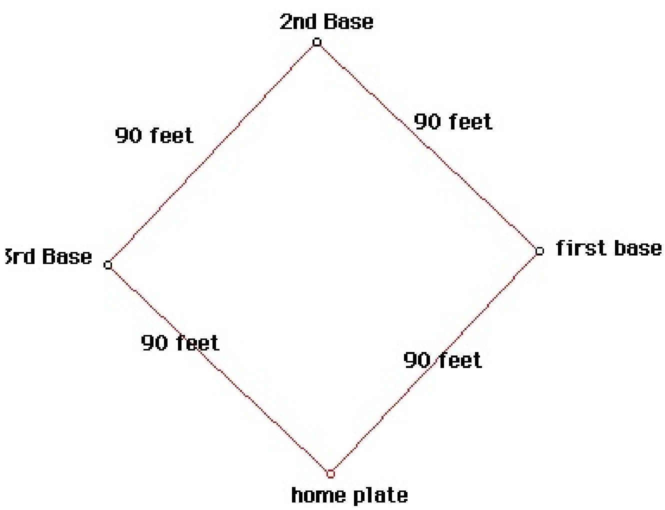


Figure 3: Baseball "Diamond"

Theorem and noticing that we can make two right triangle by drawing line from 2nd base to home plate we have,

$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{90^2 + 90^2}$$
$$c = 127.28 \text{ feet}$$

6 Famous Proofs of the Pythagorean Theorem

Many different proofs exist for the Pythagorean Theorem. According to Elisha Loomis there are four kinds of demonstrations for the Pythagorean Proposition. There are algebraic proofs, geometric proofs, quaternionic proofs based on vector operations and the dynamic proofs based on mass and velocity.[5] He claims that the number of algebraic and geometric proofs are limitless.[5] I have included a few taught proofs. Keep in mind that there is not only one way in proving the same illustration.

6.1 Bride's Chair

In figure 4 we have the most famous of all proofs of the Pythagorean proposition. It's the first of Euclid's two proofs(I.47). The configuration became known under a variety of names, The Bride's Chair being the most popular.

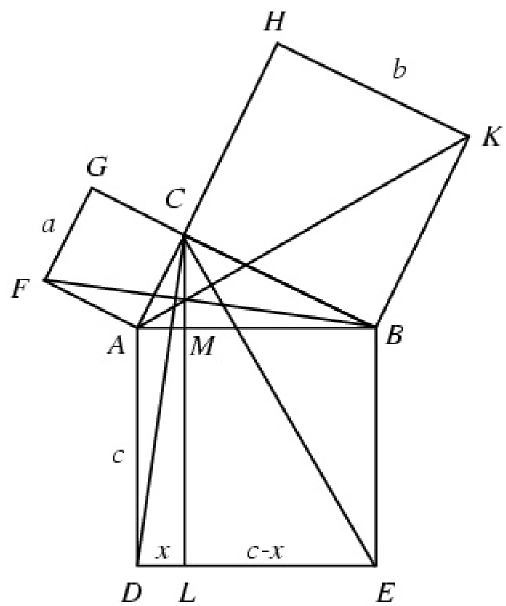


Figure 4: The bride's Chair

Let $\Delta\ ABC$ be a right triangle, $\Box\ CAFG$, $\Box\ CBKH$, and $\Box\ ABED$ be squares, and $CL \|\ BE$.. The triangles $\Delta\ FAB$ and $\Delta\ CAD$ are equivalent except for rotation, so

$$2\Delta FAB = 2\Delta CAD$$

Shearing these triangles gives two more equivalent triangles.

$$2\Delta CAD = ADLM$$

Therefore,

$$\Box ACGF = ADLM$$

Similarly,

$$\Box BC = 2\Delta ABK = 2\Delta BCE = BL$$

so

$$a^2 + b^2 = cx + c(c - x) = c^2$$

6.2 Four triangles in a square with length c

In Figure 5 we have four similar triangle and each one has area $\frac{ab}{2}$. They make a square with length c. The square has a small square in the middle with length $(a - b)$.
The area of the small square is

$$(a - b)^2$$

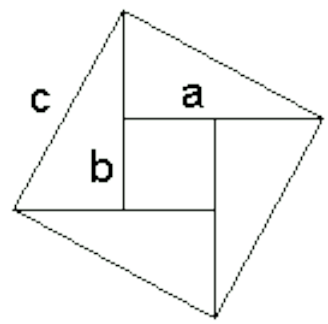


Figure 5:

and the area of the four triangles $4 \times \frac{ab}{2}$ is

$$2ab.$$

If we sum them we get,

$$\begin{aligned} &= (a-b)^2 + 2ab \\ &= a^2 - 2ab + b^2 + 2ab, \text{and} \\ c^2 &= a^2 + b^2 \end{aligned}$$

6.3 Four Triangles in a square with length$(a+b)$

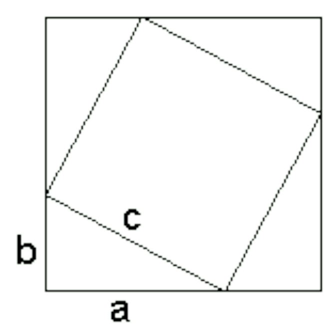


Figure 6:

In figure 6 we use the same triangles we used in the preceding proof, except that this time they form a square with length$(a+b)$ and a smaller square with length c. So calculating the area in two ways,

$$\begin{aligned} (a+b)^2 &= 4 \times \frac{ab}{2} + c^2 \\ a^2 + b^2 + 2ab &= 2ab + c^2 \\ a^2 + b^2 &= c^2 \end{aligned}$$

7 President Garfield's Proof

The following proof to the Pythagorean Theorem is from President Garfield. He discovered this proof five years before he become President. He hit upon this proof in 1876 during a mathematics discussion with some of the members of Congress. It was later published in the New England Journal of Education. The proof depends on calculating the area of a right trapezoid two different ways. The first way is by using the area formula of a trapezoid and the second is by summing up the areas of the three right triangles that can be constructed in the trapezoid. He used the following trapezoid in developing his proof

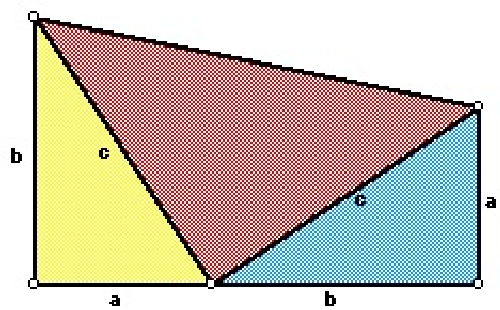


Figure 7: Garfield's trapezoid

First, we need to find the area of the trapezoid by using the area formula of the trapezoid.

$$A = \frac{1}{2}h(b_1 + b_2)$$

In Figure 7,

$$\begin{aligned} h &= a + b \\ b_1 &= a, \text{and} \\ b_2 &= b. \end{aligned}$$

The area becomes

$$\begin{aligned} A &= \frac{1}{2}(a+b)(a+b) \\ &= \frac{1}{2}(a^2 + 2ab + b^2). \end{aligned}$$

Now, let's find the area of the trapezoid by summing the area of the three right triangles. The area of the yellow triangle is

$$A = \frac{1}{2}(ba).$$

The area of the red triangle is

$$A = \frac{1}{2}(c^2).$$

The area of the blue triangle is

$$A = \frac{1}{2}(ab).$$

The sum of the area of the triangles is

$$\frac{1}{2}(ba) + \frac{1}{2}(c^2) + \frac{1}{2}(ab) = \frac{1}{2}(ba + c^2 + ab) = \frac{1}{2}(2ab + c^2).$$

Since, this area is equal to the area of the trapezoid we have the following relation:

$$\frac{1}{2}(a^2 + 2ab + b^2) = \frac{1}{2}(2ab + c^2).$$

Multiplying both sides by 2 and subtracting $2ab$ from both sides we get

$$a^2 + b^2 = c^2$$

concluding the proof.

8 Conclusions

President Garfield's life is of great admiration. He started in poverty and quickly through education was able to become all that he could be. He learned everything he could and used it to advance in life and become President of the United States. Garfield was not born, but made; and he made himself by persistent, strenuous, conscientious study and work. He was a college president, a state senator, a major general in the National army, and President of the United States.[2] No other American president had received so many rapid and varied promotions.[3] As an educator, he was, and always would have been eminently successful; he had the knowledge, the art to impart his wisdom, and the personal magnetism that impressed his love for education upon his pupils.[3] The Pythagorean Theorem is the greatest theorem in geometry. It is useful in everyday applications and it contains numerous proofs. James A. Garfield wrote his name in math books as he published one of the most taught proofs.

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