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Pythagoras: A True Renaissance Man

Anyone with any kind of mathematical background knows that it is impossible to avoid the Pythagorean Theorem. It is one of the first theorems that is taught to students in algebra or geometry and is used continually from that time forward in mathematical education. Because of the name of this theorem, any Joe Shmoe could tell you that Pythagoras had his hand in the development of the theorem, but there is more to Pythagoras than the theorem that will ensure his immortalization throughout time. Questions about Pythagoras arise as to who he was and how he lived; what made the man so to speak. Pythagoras is considered by many to be a prophet and a mystic, among other things (Boyer 47). Due to the loss of many writings during this time, Pythagoras tends to have a mysterious aura about him from a historical point of view. A thorough examination of Pythagoras's chronological life, his mythical status, his mathematical contributions, his influences, his belief systems, and his way of life ought to shed some light on the mysteries of Pythagoras.

By examining the chronology of Pythagoras, an understanding about how his life was conducted can begin to be made. There is some controversy as to exactly when Pythagoras was born, but through some reasoning and ordering of events it has been determined that he was born about the year 570 BC (Vogel 24). Pythagoras was said to have been born in Samos, an island close to Miletus, birth place of Thales (Vogel 24). When he was about 18 years old, Pythagoras started studying philosophy for about four years with three different teachers: Pherekydes of

Syros, Thales of Miletus, and Anaximander of Miletus (Gorman 24). After serving under these men's tutelage, Pythagoras traveled to Egypt and stayed there for over twenty years learning from Egyptian clergy till about 533 BC (Vogel 20). He then, as a prisoner of war, went to Babylon for a short while, learning from magicians there (Vogel 20). Pythagoras then made his way back to Samos, but not before stopping by in Sparta and Crete to study their legislation (Vogel 20). Pythagoras, in 529 BC, went to Croton on the mainland of Italy where he stayed for about twenty years (Vogel 24). In 509 BC Pythagoras left Croton to go to Metapontum where he would live out the rest of his life (Vogel 24). Pythagoras was said to have died anywhere from 503 to 490 BC, far outlasting the standard of living for that time period (Reidweg 20). All dates are approximate due to lack of good record keeping in that time. Now that the timeline of his life is known, let's dig deeper into the vagueness of Pythagoras's life.

To begin to understand Pythagoras of Samos, first it is important to understand the legends that surround him. He first and foremost was known as a mathematician and a philosopher. He also is known as an astronomer, a saint, a prophet, a performer of miracles, a magician, a charlatan, a bean hater, an animal linguist, a teacher, a cultist, and even a god incarnate (Apollo's descendent to be precise (Boyer 49; Riedweg 2, 6).)

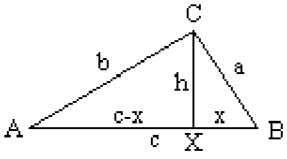
Mysteries surrounding Pythagoras's birth are told in various accounts in a legendary fashion. As almost a parallel to the birth of Jesus, Pythagoras was said to have been born from his mother under immaculate conception from the god of the sun, Apollo (Riedweg 6). There are accounts of Pythagoras having a 'golden thigh' and also having the power to be in two places at one time which gave him a divine atmosphere (Kahn 5). Like other religious figures, such as Jesus, Buddha, and even a fellow philosopher Socrates, Pythagoras was said to have not written anything himself so not to have works directly written by a divinity (Kahn 2). The 'golden thigh'

was a sign of divinity and was never seen by anyone. The primary reason that his 'golden thigh' was never seen was because under his toga, Pythagoras was said to have worn trousers, an unstylish trend of day (Riedwed 2).

This may have been one reason that people considered him a charlatan or fraud. It has been considered that the golden thigh was just a clever ruse invented by Pythagoras to elevate his status and trick people into believing he was more than he actually was. Other bits of controversial folklore include stories of him speaking the language of animals. On one account, there was a bear terrorizing a neighborhood and Pythagoras seized the bear, fed it fruit, and then convinced it to never attack another living creature (Riemweg 2). It is because of stories such as these that a clear perception of Pythagoras cannot be made from the early writings about him since many are controversial and seemingly untrue. With this in mind, an examination of Pythagoras in the world of mathematics is the next mask to be unveiled.

As with any person, it is best to start at the end so one can understand where the story is headed. Pythagoras had many contributions to the world, but most notable are his works in mathematics. It is hard to distinguish between what was done solely by Pythagoras and what was done by his followers, the Pythagoreans. Because of this, for the duration of the paper, when something is attributed to Pythagoras, it is to be understood that it may have been discovered by the Pythagoreans as well since in that time period it was common practice that most students attributed their work to their master; in this case Pythagoras (Boyer 48).

[1]The Pythagorean Theorem states: In a right triangle, with sides (legs) a and b, and hypotenuse c, then c_=a_+b_. A right triangle is a triangle with one right angle (an angle of 90°). Its hypotenuse is the side opposite the right angle.
Proof : a simple proof is an algebraic proof using similar triangles ABC, CBX, and ACX (in the diagram): Since corresponding parts of similar triangles are proportional, a/x=c/a or a_=cx. And b/(c-x)=c/b or b_=c_-cx or c_=cx+b_. Substituting a_ for cx, we get c_=a_+b_.



[1] http://www.jimloy.com/geometry/pythag.htm - Proof of the Pythagorean theorem

The reason the Pythagorean Theorem is so closely related to the name of Pythagoras is unknown, but there are many speculations as to why. Pythagoras, having studied under the Babylonians, would have known about what are known today as Pythagorean triples since the Babylonians had tables of these triples of numbers (Kahn 32). It has been asserted that Pythagoras may have just introduced these triples to Greece, so the people would assume he came up with the idea (Kahn 32). Another theory is that the simplest triple, 3-4-5, can be used in the mystical number system that Pythagoras believed in to show how 3, a male, and 4, a female, could come together and make 5, the number of marriage (Kahn 33). Perhaps the simplest answer though is that Pythagoras showed a simple intuitive proof of the theorem such as the special case of a square with the square's diagonal as an isosceles right triangle (Kahn 33). This, along with his predisposed knowledge of the Babylonian triples, might have led Pythagoras to come up with a preliminary theorem that, through further revisions and proofs, led to the Pythagorean Theorem that is known today.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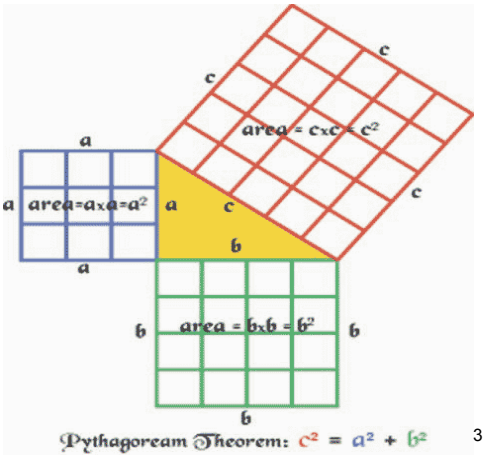

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Another area in which Pythagoras dabbled was music. Pythagoras was said to be one of the first to do any kind of work with mathematics in music (Riedweg 27). The primary foundations of music are seen in mathematics, most notably in the intervals of an octave, a fourth

[2] http://math152.files.wordpress.com/2008/11/plimpton-322.jpg - Babylonian table of triplets of numbers
[3] http://www.myastrologybook.com/PythagoreanTheoremProof.gif - Pythagorean theorem 3-4-5

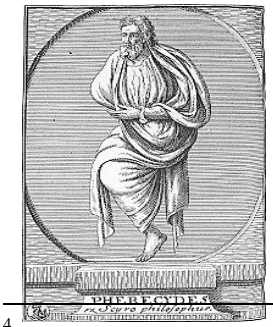
note, and a fifth note, created by making the string length half as long, a third as long, and a fourth as long respectively (Riedwed 27). Works that stemmed from this simple observation of Pythagoras influenced music for many centuries. Until Ptolemy wrote *Harmonics* in the second century AD, practically all works on music and its harmony had heavy references to Pythagoras and his work with concords (Kahn 155). Ptolemy made corrections to mistakes that Pythagoras had in his original work, but he still attributed many of his findings to the 'empirical observations of Pythagoras himself (Kahn 155).' The influence of Pythagoras's work in music can even be seen in the Renaissance period with Pythagorean intonation, which are ratios such as 3:2 and 4:3 that describe specific notes (Kahn 156).

As fore mentioned, Pythagoras spent some time studying governments and legislation in Sparta and Crete. Upon Pythagoras's arrival in Croton, he began to speak publically to men and women alike (Riedweg 12). At first he just spoke at gymnasiums and public areas, but soon he was approached and praised by local officials saying he was "…able to find the kind of wisdom digestible by each individual… (Riedweg 12)." The leaders of Croton asked Pythagoras to let them know if he had any suggestions for the community and that's precisely what he did. Pythagoras proceeded to give a list of decrees to the Croton leaders which strongly resemble teachings of the Christian persuasion (Riedweg 14). These ideas that Pythagoras had for Croton's leaders included putting others before one's self, the repudiation of oaths to the gods, in essence turning the other cheek, and even for the citizens to remain faithful to their spouses; a surprising assertion to ask of the people of that ancient time (Riedweg 14). These speeches and proposals given to the leaders of Croton reveal one of the less familiar images of Pythagoras; his role of a moral influence that expanded beyond his own pupils to the residents of Croton and

surrounding areas (Riedweg 16). Pythagoras especially talked about the importance of education, for it is what 'separates men from animals (Riedweg 13).'

Pythagoras's popularity grew almost instantly; his charismatic presence seemingly overwhelming to the Croton society (Riedweg 12). Soon Pythagoras and his followers, with numbers growing to over 300, were taking control of the legislation at Croton (Vogel 189). "…[T]hey governed the state excellently, so that its constitution was in effect a true aristocracy (Vogel 189)." Pythagoras used his legislative influence to implement equality for men and women into the society (Riedweg 15). The reform to the society of that day was immense and led to extreme popularity for Pythagoras and his followers.

Pythagoras also had a hand in astronomy. With Pythagoras's belief that the number ten was a perfect number, he sought to create a model of the heavenly bodies into a cosmos (Kahn 159). Since there were only seven know planets, Pythagoras surmised that there must be three more celestial bodies: the great fire, the sphere of the fixed stars, and the counter earth (Boyer 54). Not including the great center fire, Pythagoras claimed that the remaining nine heavenly bodies paralleled the nine muses, making what he calls the harmony of the cosmos (Riedweg 13). Though today we know that the 'cosmos,' our solar system, is not as Pythagoras described, it did lead to others such as Copernicus and Johannes Kepler making significant discoveries in the field of astronomy. (Kahn 161).With all of his contributions in so many areas, Pythagoras must have had mentors from all different areas of mathematics.



[4]

There are many influential people that directly affected the life of Pythagoras. One such man is Pherekydes of Syros. Pherekydes was most likely the chief influence on Pythagoras's views of the world (Gorman 25).

[4] http://upload.wikimedia.org/wikipedia/commons/thumb/7/75/Pherecydes.jpg/200px-Pherecydes.jpg - Pherekydes of Syros

Pherekydes was said to have introduced the idea of reincarnation to Pythagoras which he derived from the sacred books of the Phoenicians and their belief in the immortality of the consciousness (Gorman 26). This led to the later dogmatic belief of Pythagoras on the transmigration of souls and knowledge gained through the unseen world of numbers (Gorman 26). Pherekydes was said to have noticed that Aithalides was a former life of Pythagoras, stimulating Pythagoras into remembering his past life, making it easier for Pythagoras to notice his other past lives (Gorman 26). Together they even determined that there were 216 years between each of their reincarnations (Gorman 29). This number, 216, even held significance in the world of numbers. Not only was six the first perfect number, having all its proper factors added together to get itself back, but it is also a circular number, that is all positive integer powers of six greater than one will end in a six as well (Gorman 29). A perfect number is easier to describe than a circular number. To better explain a circular number here is an example: $6^1 = 6$, $6^2 = 36$, $6^3 = 216$, $6^4 = 1296$, $6^5 = 7776$, $6^6 = 46676$. Notice that the last digit in each number is a six. This works for all positive integer powers of six. Pythagoras ultimately nursed his dying master while he was fading till his death in Delos (Gorman 25).

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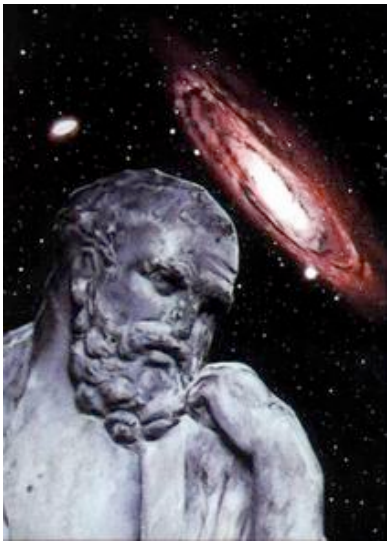

Another philosopher that Pythagoras engaged with is Anaximander. The main idea in which Anaximander is said to have passed on to Pythagoras is the idea that there is an infinite void beyond the universe (Gorman 32). Often Anaximander's contributions to the development of the young Pythagoras stop there, but he offered other things to the

[5] https://reich-chemistry.wikispaces.com/file/view/anaximander.jpg - Anaximander

philosopher-to-be as well. Anaximander was said to be the first Greek to construct horoscopes, an art learned from the Syrians and the Babylonians (Gorman 32). This is quite possibly the reason that Pythagoras took his trip later on in life to Egypt and Babylon. With limited of knowledge about the subject of astronomy, there is no doubt that Pythagoras could not resist traveling to a foreign country to fill the gap in his knowledge about the stars and other celestial beings. Anaximander combined the ideas that there are infinitely many solar systems and that there is an infinite amount of space to conclude that though there was no destruction of souls. There were never repeated souls in the same solar system (Gorman 33). His thoughts on the subject were that there are infinitely many solar systems which meant that there is an infinite number of souls in the universe (Gorman 33). These souls, however, are not limited to be reincarnated within their own solar system (Gorman 33). By sheer probability alone, the chances that a soul would be reincarnated in the same solar system twice would be slim to none. Obviously Pythagoras has different views than Anaximander on this issue because he believed himself to be reincarnated from a former human on earth. Though they differ in this view, Pythagoras did take the belief that there is an infinite amount of space, but that each solar system recycled its own souls (Gorman 31). Anaximander did extensive work on the cosmos and how it was constructed, too. He, again, had a differing view from Pythagoras in that Pythagoras used his view of there being a finite amount of souls to be in opposition with infinite amounts of time to say that there is a re-using of souls over and over thus there are conflicting powers between the finite and the infinite (Gorman 33). Opposition was a primary aspect of Pythagoras's way of life. Some other things that were in the list of opposites were odd and even, one and many, right and left, male and female, light and darkness, and square and oblong (Gorman 141). Anaximander, however, feels that these things are not conflicting, but rather working in harmony

(Gorman 33). Though Anaximander and Pythagoras didn't agree on every aspect of the construction of the cosmos, Pythagoras did adopt the idea of an infinite universe, though his work in astronomy and cosmology was strictly limited to our own solar system. Anaximander died in 547-546 BC when Polycrates the tyrant was at the pinnacle of his power (Gorman 31).

[6]

Thales of Miletus was the third philosopher in Ionia in which Pythagoras studied under. When Thales and Pythagoras met, Thales was already an old man (Gorman 35). Thales was thought to be the one to suggest that Pythagoras go to Egypt to learn the ideas that he did while he was a youth because he was too old to teach him all that he knew before he died (Gorman 35). Though it is not known exactly what Thales taught Pythagoras, his lingering presence can be seen through certain ideals that both the philosophers believed in, not to mention that Thales most likely introduced Pythagoras and Anaximander (Gorman 35). One thing in particular that Thales is thought to have taught Pythagoras was that everything in the universe was animate (Gorman 35). Thales came to this conclusion by dealing with lodestone and amber. The inherent forces in these two stones could attract other objects in a magnetic fashion, therefore, to Thales. This means that even lifeless rocks have psychic existence (Gorman 36). Both Thales and Pythagoras agreed that not only was everything animate, but also that everything was filled with gods, which is where Pythagoras came up with his theories on number mysticism (Gorman 35). Pythagoras believed in number mysticism in the form of numerology (Bowyer 52). Later the Pythagoreans based aspects of their philosophy and way of life on number mysticism (Boywer 52). Like the other philosophers, Thales, too, taught Pythagoras about the immortality of the soul and the possibility

[6] http://www.britannica.com/blogs/wp-content/uploads/2007/10/thales.jpg - Thales of Miletus

of reincarnation (Gorman 36). Pythagoras left Thales when he was about twenty-two to go to Egypt, and Thales undoubtedly died while Pythagoras was there (Gorman 38). Through these three men, four of the main elements of Pythagoras's teachings are seen: reincarnation, eternal recurrence, immortality of souls, and that all life is related.

Pythagoras, upon coming to Croton, almost immediately began to have a following. Under the tutelage of Pythagoras, the undisputed leader of this group, Pythagoras's students were aptly named the Pythagoreans. As seen from the way that Pythagoras 'ran' the Croton legislator, these men were interested in upholding themselves to the highest moral standards (Riedweg 14). In this, for lack of a better term, cult, there were strict rules that had to be obeyed or else expulsion from the society. To get into this secret society, first one had to be initiated. The initiation sentence for getting into the secret society is a five year period of silence (Gorman 189). Through these five years the practices of the society are to be learned and perfected so that upon finally being considered a member of the society one does not get banished for not following one of the rules. As a communist community of sorts, upon entering into the society, the new member's personal possessions are reassigned to the society as communal assets (Gorman 189). Once inducted into the society, you are cultivated into a binding friendship that is unconditional (Riedweg 39). The silence for five years indicates a major characteristic of the Pythagoreans: they are a *secret* society (Gorman 189). In large, secrecy is contained within the society through the strong friendships with other members of the society, and therefore a certain disdain for outsiders (Riedweg 39). Coupled with the self reflecting practices twice a day for ethical purification and strengthening the memory, being a close knitted society awards its own faults (Riedweg 33). Due to the self reflections performed every morning, in which the Pythagoreans worked on their memory by first recalling everything they did the day before, and

then trying to perfectly recite in their heads the conversations they engaged in the previous day, the Pythagoreans were confident in their abilities to use oral teaching as their sole source of instruction (Riedweg 33; Kahn 21). Because of their strife in trying to keep the contents of their society a secret, the Pythagoreans did not keep any written records (Kahn 21). This leads to large gaps in history about the Pythagoreans because the only people that knew completely about their ways of life are now long gone so we must use what information that we have to construct the structure of the Pythagorean society.

The Pythagoreans share a common religious belief. It is the belief in reincarnation of the soul (Boyer 48). As stated before, according to the Pythagoreans, all things have souls and when they 'die' their souls are reincarnated as either an animal, plant, or human (Gorman 36). Conveniently enough, this belief fits perfectly into the moral standards of the Pythagoreans. With this belief the Pythagoreans can not only justify not harming creatures morally, but religiously as well. Pythagoreans logically enough do not condone anyone hurting any animal because otherwise a friend from a former life that might reside in the animal being hurt will incur unprovoked pain and weighed on their consciences (Boyer 48). Naturally this leads right into the dietary choices of the Pythagoreans. It is for this reason, along with the health factor, that the Pythagoreans all enjoin into a life of vegetarianism (Boyer 48). Another stipulation on their diet is that they are not allowed to eat beans (Riedweg 36). This restriction is much less apparent than the pledge to not eat meat, but the members of the society didn't take this lightly.

An anecdote about Pythagoreans and a bean field is the only way to accurately portray their level of commitment for all the laws of the society, even the ones that seem to have no real backing.

> "One time, when the Pythagoreans were being persecuted, a group of royal guards were sent to kill a group of Pythagoreans at their homes. When they found out that the guards were coming they took off toward a field, for though courage is a facet that Pythagoreans value, they believe knowing when to run is a large part of courage. When they reached the field they realized that it was a bean field so they stopped and armed themselves with sticks and stones against the armed guards. They were easily pacified and two were taken as prisoners. When the tyrant asked them why their friend died rather than run through the bean field, the male prisoner said, "Those people accepted death rather than tread on beans, but I would rather tread on beans than betray to you the reason why." He was executed and his wife was tortured into revealing the secret of the beans. The woman bit off her tongue before they could get the information out of her (Riedweg 39)."

The Pythagoreans level of commitment to their sect was so great that they would knowingly die or be tormented before going against their code of conduct.

Other things were required of the Pythagoreans. One thing that the Pythagoreans do is play music to heal their souls by being in tune with the harmonics of the cosmos (Riedweg 30). In the mornings, after they had their breakfast which consisted of only bread and honey, the Pythagoreans went in groups of two or three to seek out aesthetically beautiful places so they may start their day with spiritual restoration (Riedweg 30). Perhaps the most fundamental belief of the Pythagoreans is to search out philosophical and mathematical understanding, so that they might increase their knowledge as a moral basis for the conduct of life (Boyer 48). Through this desire to have understanding of math and philosophy, the Pythagoreans developed the terms abundant, deficient, and perfect numbers (Boyer 55). Abundant, deficient, and perfect numbers

are related to the sum of the non trivial factors of the original number. An example of these would look like this:

Abundant: 36 : 36, 13, 12, 9, 6, 4, 3, 2, 1 $36<13+12+9+6+4+3+2+1=50$

Deficient: 7: 7, 1 $7>1$

Perfect: 6: 6, 3, 2, 1 $6=3+2+1=6$

In their never-ending search for knowledge, the Pythagoreans stumbled upon the figurative numbers (Bowyer 54). Among the figurative numbers were the triangular numbers, oblong numbers, and square numbers. Each of these different types of figurative numbers can be described by the sum of a series of numbers.

Triangular Numbers: $N= 1 + 2 + 3 + \ldots + n = n(n+1)/2$

Square Numbers: $N= 1 + 3 + 5 + \ldots + (2n-1) = n^2$

Oblong Numbers: $N = 2 + 4 + 6 + \ldots + 2n = n(n+1)$ (Bowyer 54)

The Pythagoreans also implement their number mysticism into all ways of their life. According to their particular beliefs on number mysticism, the Pythagoreans consider the number ten to be the divine number and created certain models, such as the cosmos, with some relation to that number (Boyer 53).

Pythagoras, as the spear head of his society, was successful. He makes an appearance in math books, but he also is cited in practically every simple harmonics book written since his time. He even makes appearances in astronomy books. So whether the man had a golden thigh, was a reincarnated soldier from the Trojan War, was immaculately conceived by Apollo himself, or was just the biggest quack of all times, Pythagoras made a huge impression on the world through his mythical status, his mathematical contributions, his influences, his belief systems, and his extreme way of life.

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