# Chapters 5 and 6

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# Anaxagoras

500-428 B.C.

– He was more of a natural philosopher than a mathematician.
– Said that the sun was a “fiery rock”
– His two famous theories were of Nutrition and the Postulation of the Mind

# The three famous problems

#1 Trisecting an Angle

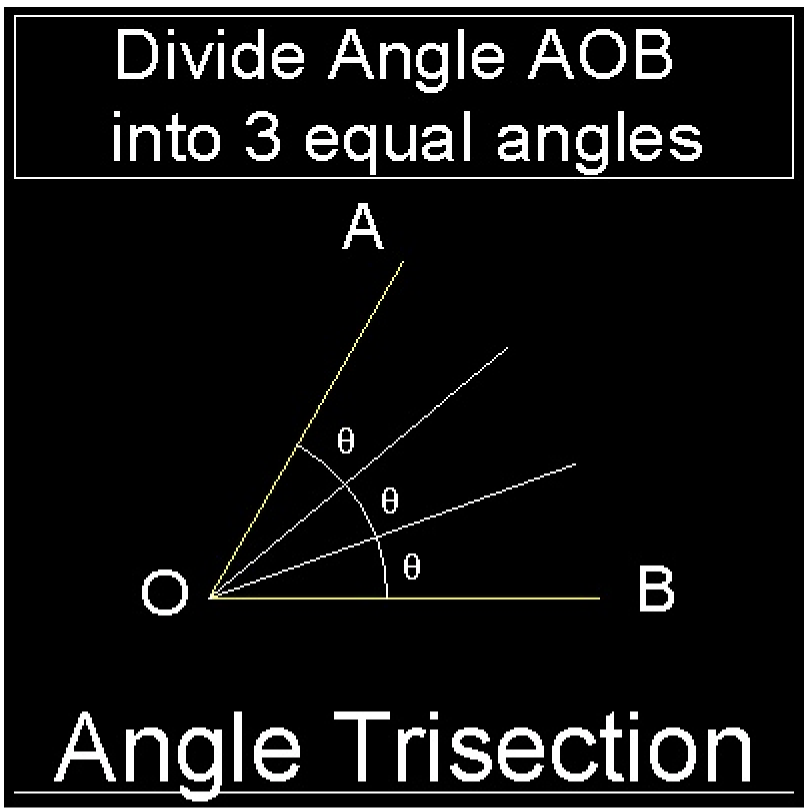
#2 Doubling a Cube

#3 Squaring the Circle

All 3 problems are unsolvable with only a compass a straightedge.

# Problem #1

Divide and arbitrary angle into three angles.



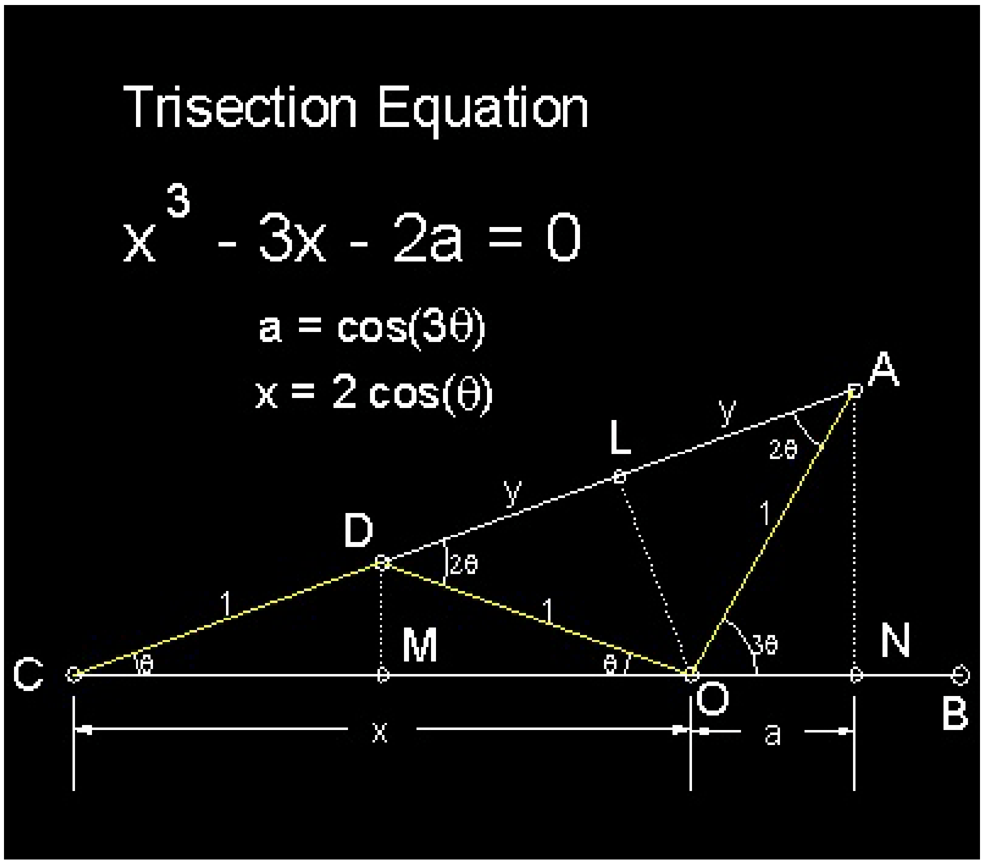

The origin of the problem:

It was easy to cut an angle into even halves, but it was hard to cut it into three.

Note: They were only were able to use a straightedge and a compass.

Only angles 45, 72, 90, & 180 could be done, and with their tools, there were no other possibilities.

# Solution




Since three triangles CDM, COL and CAN are similar.

CM/CD = CL/CO = CN/CA.

So:
x/2 = (1 + y)/x = (x + a)/(1 + 2y)

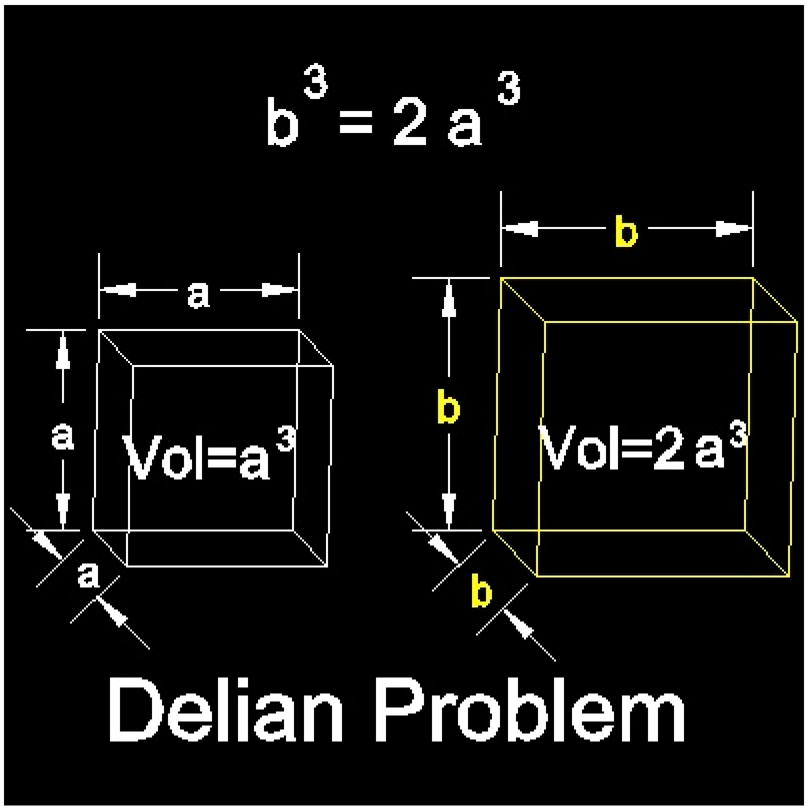
which gives:
x2 = 2 + 2y and
1 + 2y = 2(x + a)/x

Eliminating y from these two equations, we obtain
x 2 - 1 = 2(x + a)/x or

**x 3 - 3x - 2a = 0**
where **x = 2cos(_)** and
**a = cos(3_)**

# Problem #2

Construct the edge of a cube so that it is double the volume of the other cube.




Origin: To rid the plague, the Greeks needed to double the volume of the solid altar.

*They could only use a straightedge and a compass

# Solution

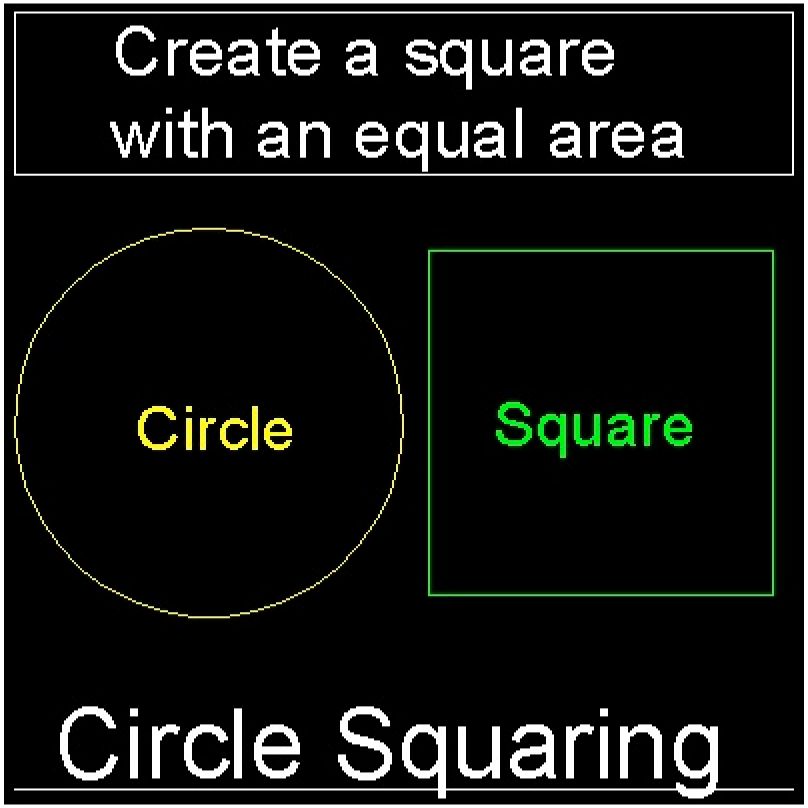
Hippocrates of Chios discovered that the duplication of the cube is equivalent to finding two mean proportionals in continued proportion between two given straight lines.

That is, if a/x = x/y = y/b
where a and b are given,
then (a/x)3 = a/b
Therefore if a = 1 and b = 2, x = (2)1/3
So we have cubic root of 2.

# Problem #3

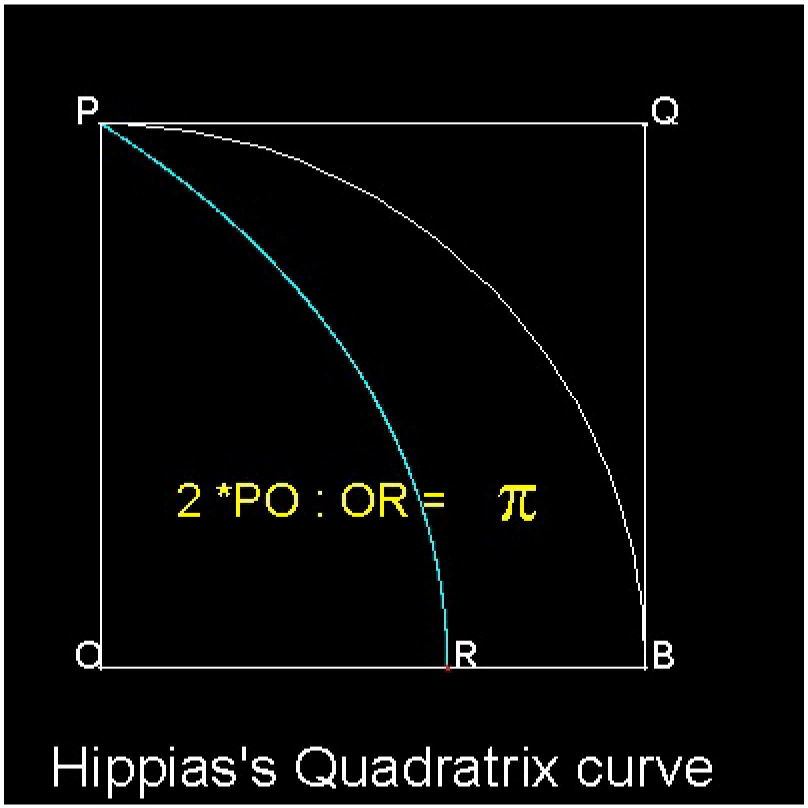
Construct a square with the same area of a circle.

*The is equivalent to constructing a line segment of the length of pi or its square root.




Origin: Men wanted to find the circumference of a circle.

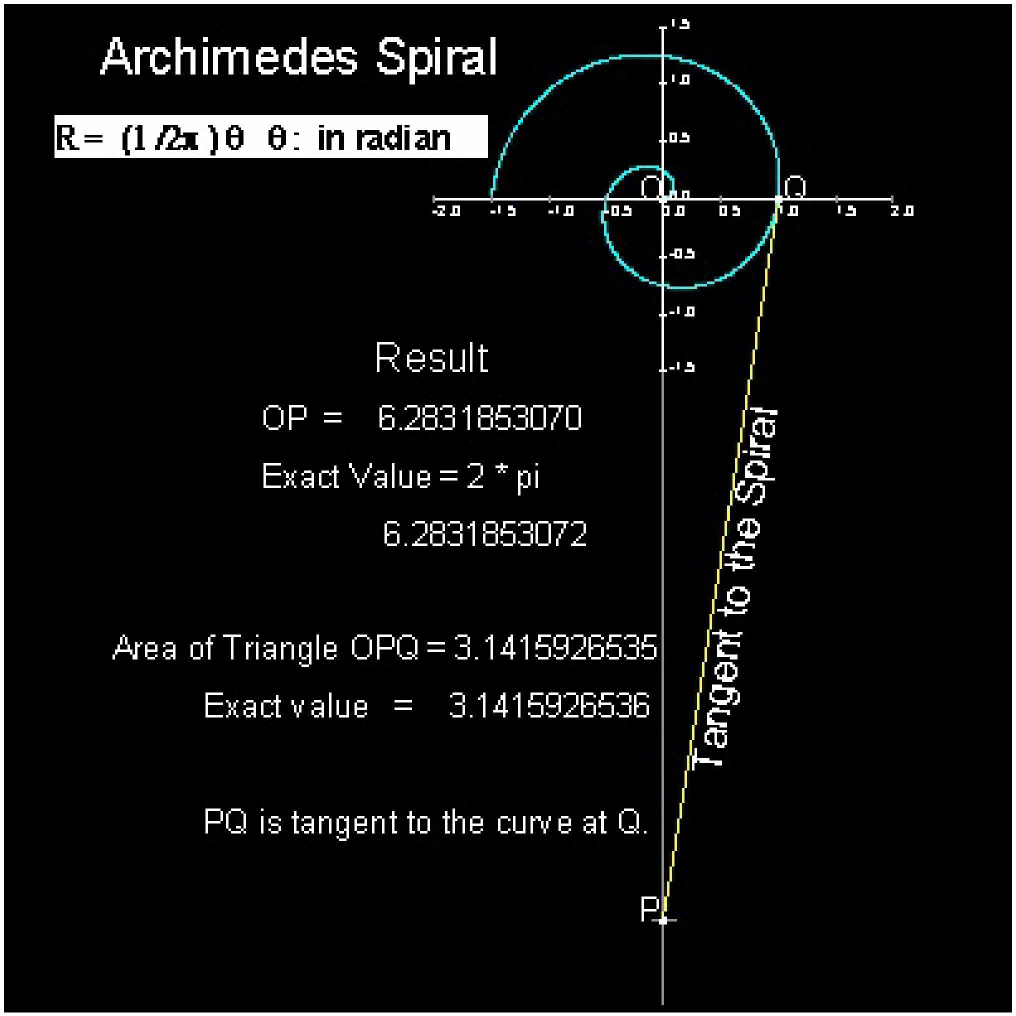
# Solution




Hippias’ invented a curve now called "Quadratrix", because it can be used for rectifying a circle.

But Hippias invented this curve for Angle Trisection, and called it "Trisectrix".

# Archimedes' Spiral




Conon of Samos (about 280BC - about 220 BC) invented a spiral, the polar equation of which is written as _ = a _.
This curve was used by Archimedes (287BC - 212 BC) for squaring the circle.

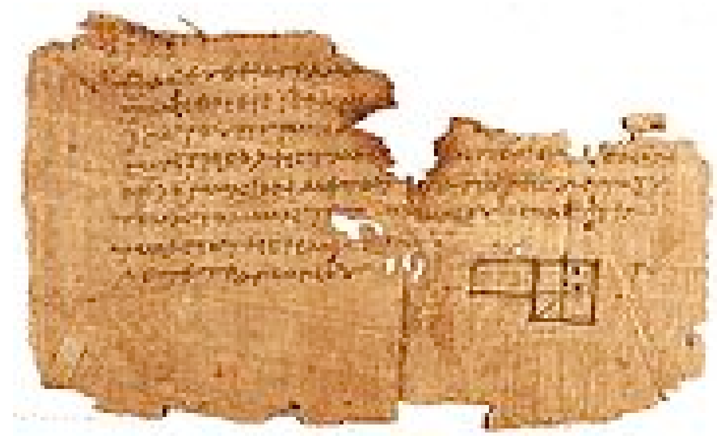
# Hippocrates "Elements of Euclid"


Copy of the first
English version

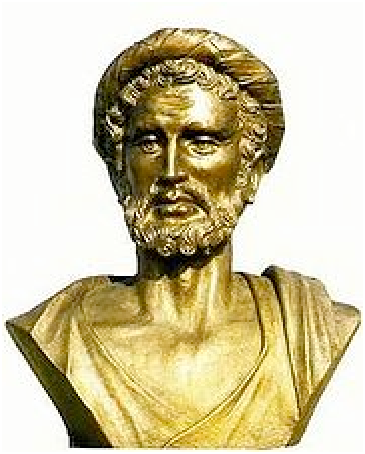
Book 1-4 Plane Geometry

Book 5-10 Ratio's and Proportions

Book 11-13 Spatial Geometry



Proposition 5 of Book 2



# Archytas

*Believed to be the founder of Mathematical Mechanics

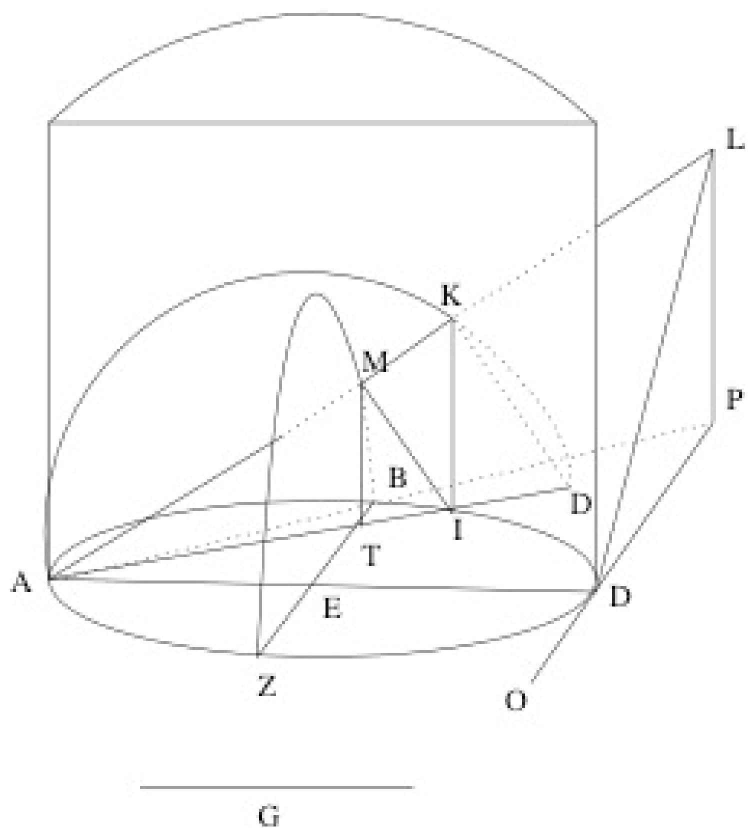
*He was reputed to have designed and built the first artificial, self-propelled flying device, a bird-shaped model propelled by a jet of what was probably steam, said to have actually flown some 200 meters. (It was called the Pigeon)

*Solved the problem of doubling the cube in his manner with a geometric construction. (Example on next slide)

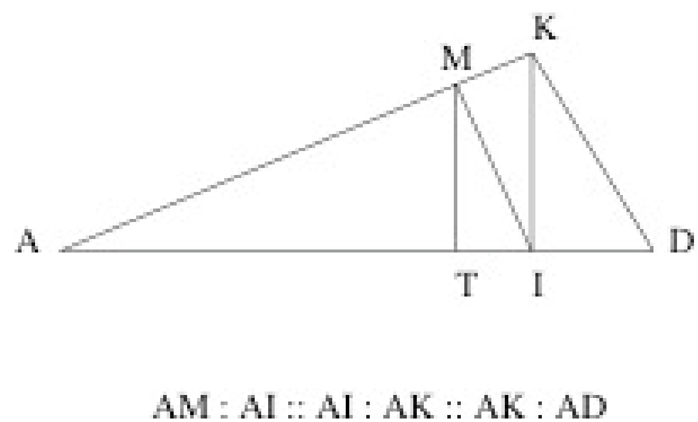
*Constructed the music theory: ratio $(n+1) : n$ (If the two string lengths are in the ratio 4 : 3, we will hear a fourth, and, if the ratio is 3 : 2, we will hear a fifth. )

*7 Liberal arts

# Doubling the Cube




The **Archytas Curve** is created by placing a semicircle (with a diameter of d) on the diameter of one of the two circles of a cylinder (which also has a diameter of d) and then rotating the semicircle about the cylinder's diameter. This rotation will cut out a portion of the cylinder forming the Archytas Curve.



# Incommensurability

Generally, two quantities are **commensurable** if both can be measured in the same units. For example, a distance measured in miles and a quantity of water measured in gallons are *in*commensurable (thus stressing the point that they cannot rationally be compared)

(Show Proof)

# Zeno’s Paradoxes

***In a race, the quickest runner can never overtake the slowest, since the pursuer must first reach the point whence the pursued started, so that the slower must always hold a lead.***

In the paradox of Achilles and the Tortoise, Achilles is in a footrace with the tortoise. Achilles allows the tortoise a head start of 100 feet. If we suppose that each racer starts running at some constant speed (one very fast and one very slow), then after some finite time, Achilles will have run 100 feet, bringing him to the tortoise's starting point. During this time, the tortoise has run a much shorter distance, for example 10 feet. It will then take Achilles some further time to run that distance, in which time the tortoise will have advanced farther; and then more time still to reach this third point, while the tortoise moves ahead. Thus, whenever Achilles reaches somewhere the tortoise has been, he still has farther to go. Therefore, because there are an infinite number of points Achilles must reach where the tortoise has already been--he can never overtake the tortoise.

***That which is in locomotion must arrive at the half-way stage before it arrives at the goal.***

$$\left\{\cdots,\frac{1}{16},\frac{1}{8},\frac{1}{4},\frac{1}{2},1\right\}$$

This description requires one to complete an infinite number of tasks, which Zeno maintains is an impossibility.

This sequence also presents a second problem in that it contains no first distance to run, for any possible (finite) first distance could be divided in half, and hence would not be first after all. Hence, the trip cannot even begin. The paradoxical conclusion then would be that travel over any finite distance can neither be completed nor begun, and so all motion must be an illusion.

This argument is called the *Dichotomy* because it involves repeatedly splitting a distance into two parts. It contains some of the same elements as the *Achilles and the Tortoise* paradox, but with a more apparent conclusion of motionlessness.

***If everything when it occupies an equal space is at rest, and if that which is in locomotion is always occupying such a space at any moment, the flying arrow is therefore motionless.***

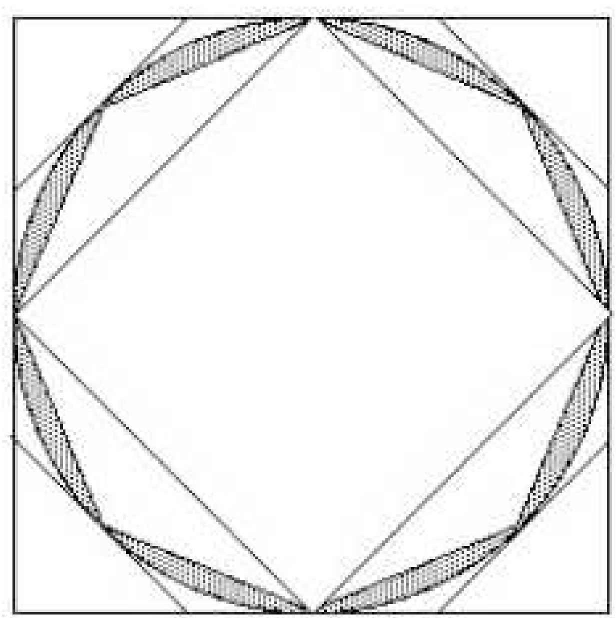
In the arrow paradox, Zeno states that for motion to be occurring, an object must change the position which it occupies. He gives an example of an arrow in flight. He states that in any one instant of time, for the arrow to be moving it must either move to where it is, or it must move to where it is not. It cannot move to where it is not, because this is a single instant, and it cannot move to where it is because it is already there. In other words, in any instant of time there is no motion occurring, because an instant is a snapshot. Therefore, if it cannot move in a single instant it cannot move in **any** instant, making any motion impossible. This paradox is also known as the **fletcher's paradox**—a *fletcher* being a maker of arrows.

Whereas the first two paradoxes presented divide space, this paradox starts by dividing time - and not into segments, but into points.

# The Quantum Zeno Effect

- In 1977, physicists E.C.G. Sudarshan and B. Misra studying quantum mechanics discovered that the dynamical evolution (motion) of a quantum system can be hindered (or even inhibited) through observation of the system. This effect is usually called the quantum Zeno effect as it is strongly reminiscent of (but not fundamentally related to) Zeno's arrow paradox.
- This effect was first theorized in 1958.

# The Method of Exhaustion



The area of the circle was at once greater than the area of any and all possible polygons that could be inscribed within it and smaller than the area of any and all possible polygons that could be circumscribed about it.

*This method was used by Eudoxus of Cnidos and Archimedes.

# The end of the Hellenic Period

- Marked my Alexander the Greats death(323 B.C.)
- His empire was divided
- Aristotle left Athens and died a year later

# References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- http://en.wikipedia.org/wiki/Archytas
- http://en.wikipedia.org/wiki/Zeno's_paradoxes