

Math 364 - Set Theory Problems  
Fall 2008

72. Find three sets  $A$ ,  $B$ , and  $C$  such that  $A \in B$ ,  $B \in C$ , and  $A \in C$ .

For all of the following,  $A$ ,  $B$ , and  $C$  are sets.

73. True or False and explain your reasoning.

(a) The empty set is a proper subset of every set.

(b) If  $A$  is a proper subset of  $\emptyset$ , then  $A = \{17\}$ .

(c) If  $A \subseteq B$  then  $A = B$ .

(d) If  $A = B$  then  $A \subseteq B$ .

(e) Since  $\emptyset$  is a member of  $\{\emptyset\}$ ,  $\emptyset = \{\emptyset\}$ .

(f) There is a set that is a member of every set.

**Definition 8** If  $A$  and  $B$  are sets, then the union of  $A$  and  $B$  is the set of all objects that belong to  $A$  or belong to  $B$ , denoted  $A \cup B$ . In other words,

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

**Definition 9** If  $A$  and  $B$  are sets, then the intersection of  $A$  and  $B$  is the set of all objects that belong to both  $A$  and  $B$ , denoted  $A \cap B$ . In other words,

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

74. Prove or disprove:  $\emptyset \cap A = \emptyset$  and  $\emptyset \cup A = A$

75. Prove or disprove:  $A \cap B \subseteq A$

76. Prove or disprove:  $A \subseteq A \cup B$

77. Prove or disprove:  $A \cup B = B \cup A$  and  $A \cap B = B \cap A$

78. Prove or disprove:  $A \cup (B \cap C) = (A \cup B) \cap C$  and  $A \cap (B \cup C) = (A \cap B) \cup C$

79. Prove or disprove:  $A \cup A = A = A \cap A$

80. Prove or disprove: If  $A \subseteq B$ , then  $A \cup C \subseteq B \cup C$  and  $A \cap C \subseteq B \cap C$ .

**Definition 10** Let  $A$  and  $B$  be sets. Then the complement of  $A$  relative to  $B$  is the set  $\{x \in B \mid x \notin A\}$ , denoted  $B - A$ .

**Definition 11** If for a certain problem, all of the sets being considered are subsets of a given set  $U$  then  $U$  is called a universal set.

**Definition 12** If  $U$  is the universal set and  $A \subseteq U$  then the complement of  $A$  relative to  $U$  is denoted  $A'$  and is  $U - A = A' = \{x \in U \mid x \notin A\}$

81. Prove or disprove:  $(A \cup B)' = A' \cap B'$
82. Prove or disprove:  $(A')' = A$
83. Prove or disprove:  $(A \cap B)' = A' \cup B'$
84. Prove or disprove:  $A - B = A \cap B'$
85. Prove or disprove:  $A \subseteq B$  if and only if  $B' \subseteq A'$
86. Prove or disprove:  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$  and  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ .
87. Prove or disprove:  $(A \cap B) \cup C = A \cap (B \cup C)$
88. Prove or disprove:  $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$

**Definition 13** Two sets  $A$  and  $B$  are disjoint if  $A \cap B = \emptyset$ .

89. Prove or disprove:  $A \cap B$  and  $A - B$  are disjoint.
90. Prove or disprove:  $A = (A \cap B) \cup (A - B)$ .
91. Prove or disprove:  $A - (A \cap B') = A \cap B$
92. Prove or disprove: If  $A$  and  $B$  are sets such that  $A \cup B = A \cap B$  then  $A \cap B' = \emptyset$ .
93. Prove or disprove: If  $A$  and  $B$  are sets such that  $(A \cup B)' = A' \cup B'$  then  $A = B$ .
94. Prove or disprove: Let  $A$ ,  $B$  and  $C$  be sets such that  $A \cup B \neq A \cap C$ . Then  $A$  is not a subset of  $C$  or  $B$  is not a subset of  $A$ .